Computer Science 50
Introduction to Computer Science I

Harvard College

Week 3

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# Divide and Conquer

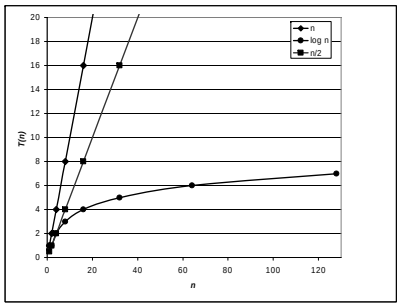



# Parallel Processing

1) Stand up.
2) Think to yourself "I am #1".
3) Pair up with someone; add your numbers together; take that sum as your new number.
4) One of you should sit down.
5) GOTO step 3 if still standing.



# Running Time
*T*(*n*)






# Running Time
*T*(*n*)

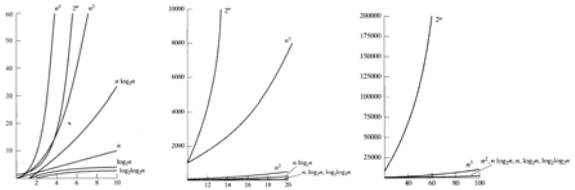
| $\log_2\log_2 n$ | $\log_2 n$ | $n$ | $n \log_2 n$ | $n^2$ | $n^3$ | $2^n$ |
|---|---|---|---|---|---|---|
| — | 0 | 1 | 0 | 1 | 1 | 2 |
| 0 | 1 | 2 | 2 | 4 | 8 | 4 |
| 1 | 2 | 4 | 8 | 16 | 64 | 16 |
| 1.58 | 3 | 8 | 24 | 64 | 512 | 256 |
| 2 | 4 | 16 | 64 | 256 | 4096 | 65536 |
| 2.32 | 5 | 32 | 160 | 1024 | 32768 | 4294967296 |
| 2.6 | 6 | 64 | 384 | 4096 | $2.6 \times 10^5$ | $1.85 \times 10^{19}$ |
| 3 | 8 | 256 | $2.05 \times 10^3$ | $6.55 \times 10^4$ | $1.68 \times 10^7$ | $1.16 \times 10^{77}$ |
| 3.32 | 10 | 1024 | $1.02 \times 10^4$ | $1.05 \times 10^6$ | $1.07 \times 10^9$ | $1.8 \times 10^{308}$ |
| 4.32 | 20 | 1048576 | $2.1 \times 10^7$ | $1.1 \times 10^{12}$ | $1.15 \times 10^{18}$ | $6.7 \times 10^{315652}$ |

TABLE 7.1 COMMON COMPUTING TIME FUNCTIONS

Figure from *C++: An Introduction to Data Structures*, by Larry Nyhoff.



# Running Time
*T*(*n*)



Figures from *C++: An Introduction to Data Structures*, by Larry Nyhoff.



# Asymptotic Notation
Informally

**O**

**Θ**

**Ω**



# Asymptotic Notation
Formally

$T(n) \in O(f(n))$

We say that the running time, $T(n)$, of an algorithm is "in big *O* of *f* of *n*" iff there exist an integer $n_0 > 0$ and a real number $c > 0$ such that $T(n) \leq c \cdot f(n)$ for all $n \geq n_0$.

$T(n) \in \Theta(f(n))$

We say that the running time, $T(n)$, of an algorithm is "in theta of *f* of *n*" iff there exist an integer $n_0$ and real numbers $c_1, c_2 > 0$ such that $c_1 \cdot f(n) \leq T(n) \leq c_2 \cdot f(n)$ for all $n \geq n_0$.

$T(n) \in \Omega(f(n))$

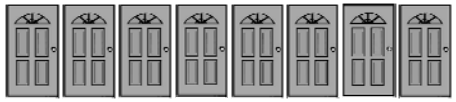
We say that the running time, $T(n)$, of an algorithm is "in omega of *f* of *n*" iff there exist an integer $n_0$ and a real number $c > 0$ such that $T(n) \geq c \cdot f(n)$ for all $n \geq n_0$.



# *O*
In English

:: $O(1)$ "constant"
:: $O(\log n)$ "logarithmic"
:: $O(n)$ "linear"
:: $O(n \log n)$ "supralinear"
:: $O(n^2)$ "quadratic"
:: $O(n^c)$ "polynomial"
:: $O(c^n)$ "exponential"
:: $O(n!)$ "factorial"

# Searching





# Linear Search

Pseudocode

```
On input n:
    For each element i:
        If i == n:
            Return true.
    Return false.
```



# Binary Search

Iterative Pseudocode

```
On input array[0], ... , array[n - 1] and k:
    Let first = 0.
    Let last = n - 1.
    While first <= last:
        Let middle = (first + last ) / 2.
        If k < array[middle] then let last = middle - 1.
        Else if k > array[middle] then let first = middle + 1.
        Else return true.
    Return false.
```



# Sum Looping

Get it?

```
int
sigma(int m)
{
    // avoid risk of infinite loop
    if (m < 1)
        return 0;

    // return sum of 1 through m
    int sum = 0;
    for (int i = 1; i <= m; i++)
        sum += i;
    return sum;
}
```

see
sigma1.c



# Sum Recursion

Get it yet?

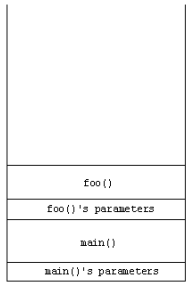
```
int
sigma(int m)
{
    // base case
    if (m <= 0)
        return 0;

    // recursive case
    else
        return (m + sigma(m-1));
}
```

see
sigma2.c



# The Stack, Revisited

Frames






# Binary Search

Recursive Pseudocode

```
On input array, first, last, and k, define recurse as:
    If first > last then return false.
    Let middle = (first + last) / 2.
    Else if k < array[middle] then
        return recurse(array, first, middle - 1, k).
    Else if k > array[middle] then
        return recurse(array, middle + 1, last, k).
    Else return true.
```



# Sorting

4 2 6 8 1 3 7 5



# Bubble Sort

Pseudocode

```
Repeat n times:
   For each element i:
      If element i and its neighbor are out of order:
         Swap them.
```

# Selection Sort
Pseudocode

```
Let i := 0.
Repeat n times:
  Find smallest value, s, between i and list's end, inclusive.
  Swap s with value at location i.
  Let i := i + 1.
```



# Sorting
Visualization



# Merge Sort
Pseudocode

```
On input of n  elements:
    If n  < 2, return.
    Else
        Sort left half of elements.
        Sort right half of elements.
        Merge sorted halves.
```



# Merge Sort
Pseudocode

$T(n) = 0$, if $n < 2$

$T(n) = T(n/2) + T(n/2) + O(n)$, if $n > 1$



# Merge Sort
How long does it take to sort 16 elements?

$T(16) = 2T(8) + 16$
$T(8) = 2T(4) + 8$
$T(4) = 2T(2) + 4$
$T(2) = 2T(1) + 2$
$T(1) = 0$



# Merge Sort

$T(1) = 0$
$T(2) = 2T(1) + 2 = 0 + 2 = 2$
$T(4) = 2T(2) + 4 = 4 + 4 = 8$
$T(8) = 2T(4) + 8 = 16 + 8 = 24$
$T(16) = 2T(8) + 16 = 48 + 16 = 64$



# Sorting
Visualization



# All Sorts of Sorts

Heap Sort
Insertion Sort
Quicksort
Radix Sort
Shell Sort
...



# Tradeoffs
Space, Time, ...

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