This is CS 50.

Harvard College's Introduction to Computer Science I

COMPUTER SCIENCE 50

WEEK 3

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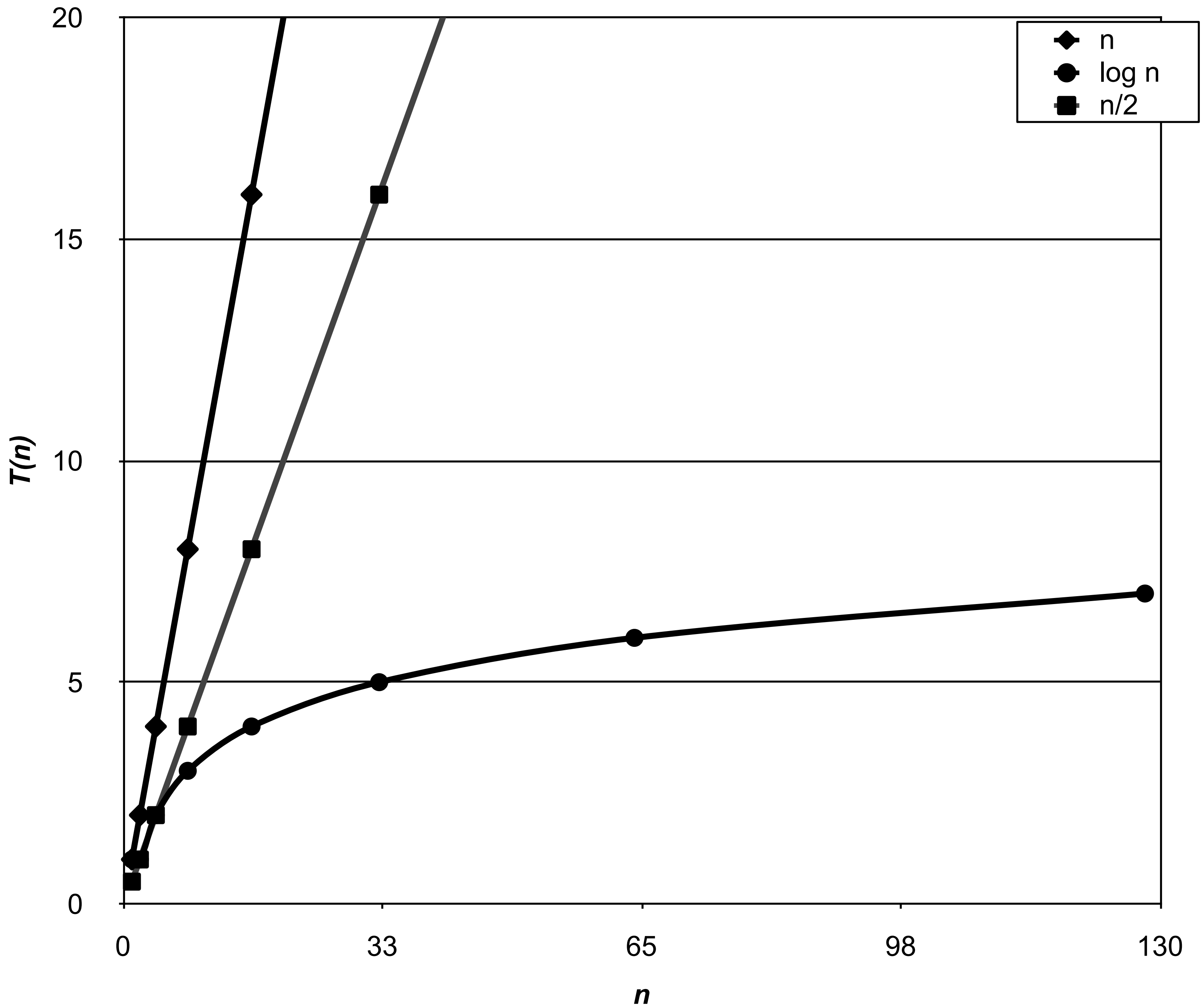
Divide and Conquer

Roll Call

1. Stand up.
2. Think to yourself “I am #1”.
3. Pair off with someone standing, add your numbers together, and adopt the sum as your new number.
4. One of you should sit down, the other should go back to step 3.

Running Time

T(*n*)



Running Time

T(*n*)

$\log_2\log_2 n$	$\log_2 n$	n	$n \log_2 n$	n^2	n^3	2^n
—	0	1	0	1	1	2
0	1	2	2	4	8	4
1	2	4	8	16	64	16
1.58	3	8	24	64	512	256
2	4	16	64	256	4096	65536
2.32	5	32	160	1024	32768	4294967296
2.6	6	64	384	4096	2.6×10^5	1.85×10^{19}
3	8	256	2.05×10^3	6.55×10^4	1.68×10^7	1.16×10^{77}
3.32	10	1024	1.02×10^4	1.05×10^6	1.07×10^9	1.8×10^{308}
4.32	20	1048576	2.1×10^7	1.1×10^{12}	1.15×10^{18}	6.7×10^{315652}

TABLE 7.1 COMMON COMPUTING TIME FUNCTIONS

Figure from C++: An Introduction to Data Structures, by Larry Nyhoff.

Running Time

T(*n*)

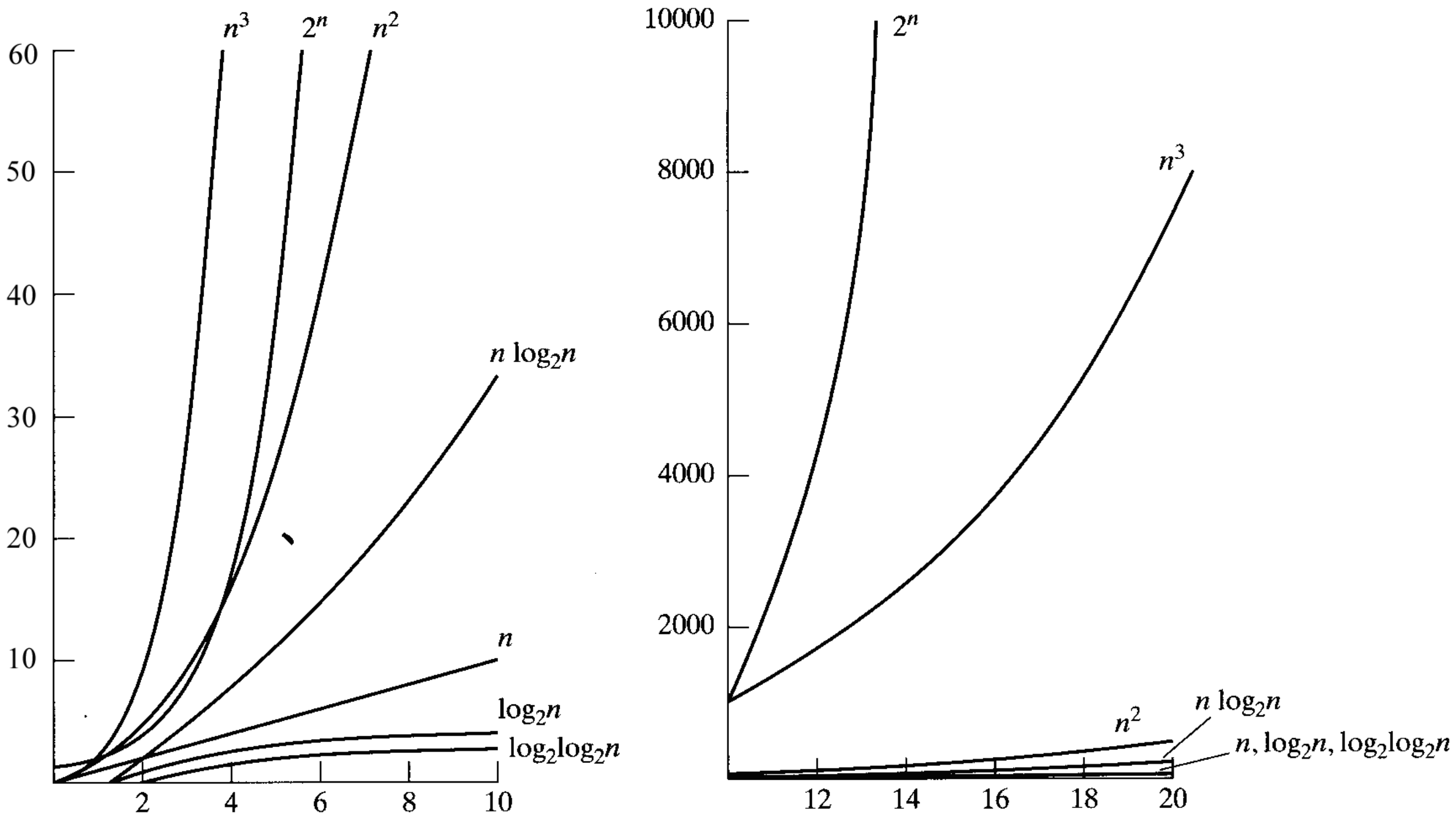


Figure from C++: An Introduction to Data Structures, by Larry Nyhoff.

Asymptotic Notation

Informally

$$O$$

$$\Theta$$

$$\Omega$$

Asymptotic Notation

Formally

$T(n) \in O(f(n))$

We say that the running time, $T(n)$, of an algorithm is "in big O of f of n" iff there exist an integer $n_0 > 0$ and a real number $c > 0$ such that $T(n) \leq c \cdot f(n)$ for all $n \geq n_0$.

$T(n) \in \Theta(f(n))$

We say that the running time, $T(n)$, of an algorithm is "in theta of f of n" iff there exist an integer n_0 and real numbers $c_1, c_2 > 0$ such that $c_1 \cdot f(n) \leq T(n) \leq c_2 \cdot f(n)$ for all $n \geq n_0$.

$T(n) \in \Omega(f(n))$

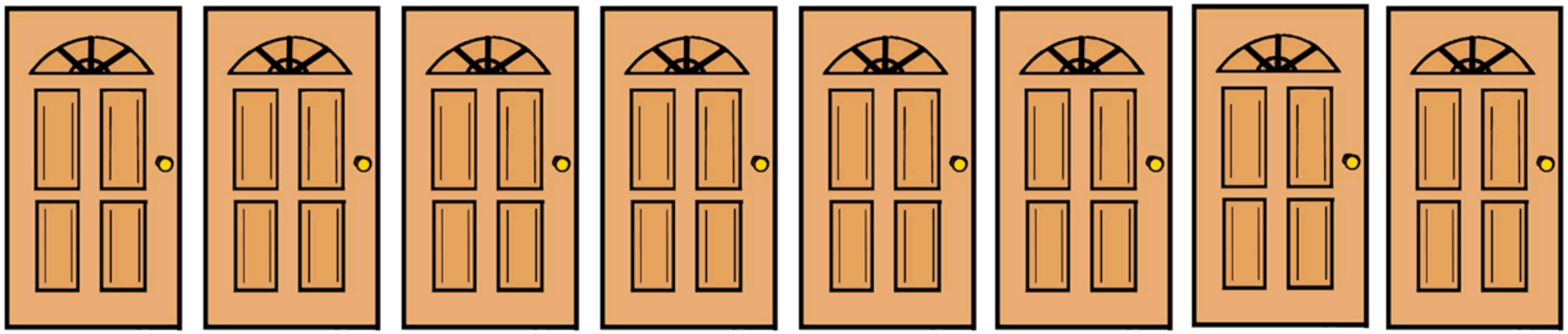
We say that the running time, $T(n)$, of an algorithm is "in omega of f of n" iff there exist an integer n_0 and a real number $c > 0$ such that $T(n) \geq c \cdot f(n)$ for all $n \geq n_0$.

O

In English

$O(1)$	"constant"
$O(\log n)$	"logarithmic"
$O(n)$	"linear"
$O(n \log n)$	"supralinear"
$O(n^2)$	"quadratic"
$O(n^c)$	"polynomial"
$O(n!)$	"factorial"

Searching



Linear Search

Pseudocode

```
On input n:
    For each element i:
        If i == n:
            Return true.
    Return false.
```

Binary Search

Iterative Pseudocode

```
On input array [0], ... , array [n – 1] and k:
    Let first = 0.
    Let last = n – 1.
    While first <= last:
        Let middle = (first + last ) / 2.
        If k < array [middle] then let last = middle – 1.
        Else if k > array [middle] then let first = middle + 1.
        Else return true.
    Return false.
```

Sum Looping

Get it?

```
int
sigma(int m)
{
    // avoid risk of infinite loop
    if (m < 1)
        return 0;

    // return sum of 1 through m
    int sum = 0;
    for (int i = 1; i <= m; i++)
        sum += i;
    return sum;
}
```

see
sigma1.c

Sum Recursion

Get it yet?

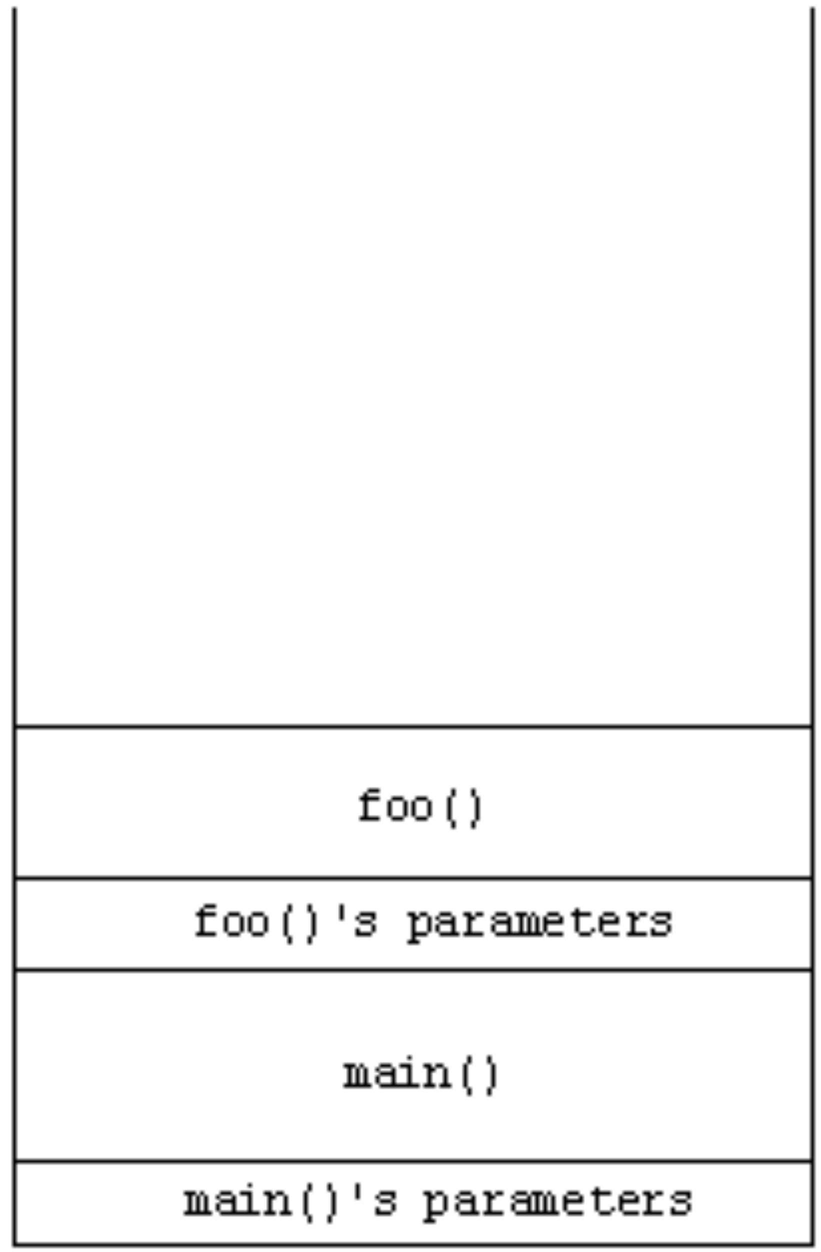
```c
int
sigma(int m)
{
    // base case
    if (m <= 0)
        return 0;

    // recursive case
    else
        return (m + sigma(m-1));
}
```

see
sigma2.c

The Stack

Frames



Binary Search

Recursive Pseudocode

```
On input array, first, last, and k, define recurse as:
    If first > last then return false.
    Let middle = (first + last) / 2.
    Else if k < array[middle] then
        return recurse(array, first, middle – 1, k).
    Else if k > array[middle] then
        return recurse(array, middle + 1, last, k).
    Else return true.
```

Sorting

4 2 6 8 1 3 7 5

Bubble Sort

Pseudocode

```
Repeat n times:
   For each element i:
      If element i and its neighbor are out of order:
         Swap them.
```

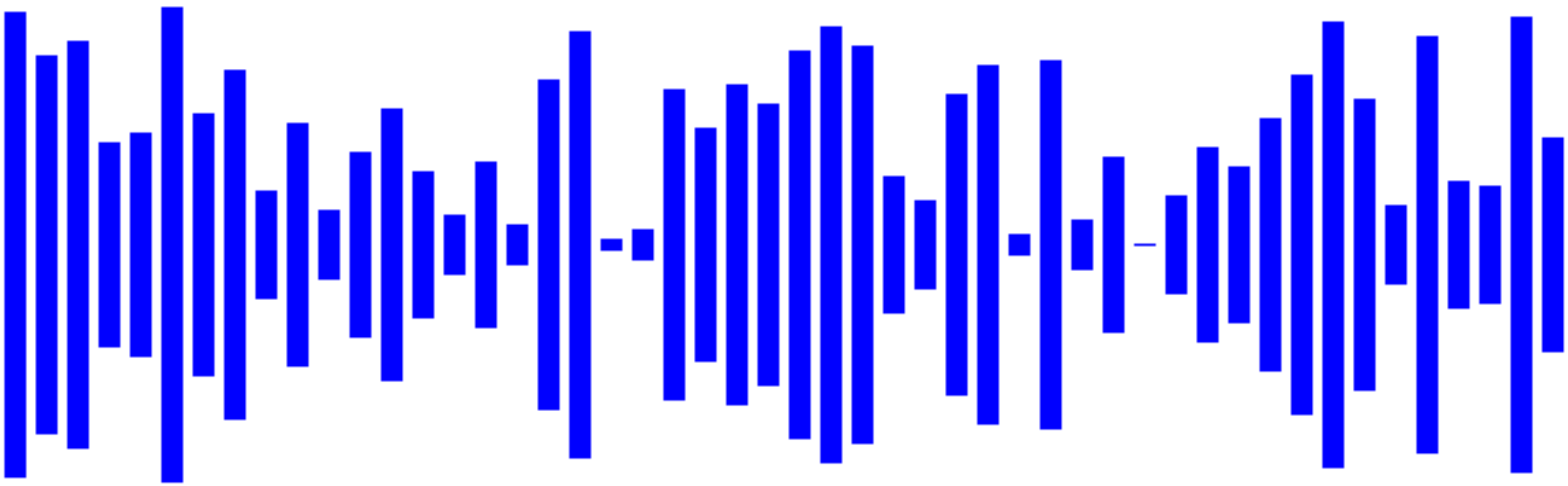
Selection Sort

Pseudocode

```
Let i := 0.
Repeat n times:
  Find smallest value, s, between i and list's end, inclusive.
  Swap s with value at location i.
  Let i := i + 1.
```

Sorting

Visualization



Merge Sort

Pseudocode

```
On input of n  elements:
    If n  < 2, return.
    Else
        Sort left half of elements.
        Sort right half of elements.
        Merge sorted halves.
```

Merge Sort

Pseudocode

$$T(n) = 0, \text{ if } n < 2$$

$$T(n) = T(n/2) + T(n/2) + O(n), \text{ if } n > 1$$

Merge Sort

How long does it take to sort 16 elements?

$$
\begin{aligned}
T(16) &= 2T(8) + 16 \\
T(8) &= 2T(4) + 8 \\
T(4) &= 2T(2) + 4 \\
T(2) &= 2T(1) + 2 \\
T(1) &= 0
\end{aligned}
$$

Merge Sort

$$T(1) = 0$$
$$T(2) = 2T(1) + 2 = 0 + 2 = 2$$
$$T(4) = 2T(2) + 4 = 4 + 4 = 8$$
$$T(8) = 2T(4) + 8 = 16 + 8 = 24$$
$$T(16) = 2T(8) + 16 = 48 + 16 = 64$$

Sorting

Visualization

All Sorts of Sorts

- Heap Sort
- Insertion Sort
- Quicksort
- Radix Sort
- Shell Sort
- ...

Tradeoffs

Space, Time, ...