## A Short Course in Numeric Self Defense

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Oil futures (2)

- How many vehicles are there?
- about one per person?
- so about 300 million

- How big is a barrel?
- bigger than a beer keg?
- like a 55 gallon drum?
- call it 50 gallons
"Perhaps the Bush administration could use the 660-billion-barrel Strategic Petroleum Reserve to push prices down."

Newsweek, 5/24/04 (when gas was under \$2/gallon)
"The average vehicle uses 550 gallons/year."
How long will the Strategic Petroleum Reserve last?


Oil futures (3)
500 gallons/year = 10 barrels/year 300 million cars $\times 10$ barrels/car $=3$ billion barrels/year

The Reserve contains 660 billion barrels. We use 3 billion barrels a year.
It will last over 200 years.

Why are we worried about oil??

Oil futures (4)
"...we said that the size of the Strategic Petroleum Reserve is 660 billion barrels.
It is actually 660 million barrels."
Newsweek, 6/7/04

Millions, billions, zillions
The bailout will cost $\$ 700$ billion, not million.
(New York Times, 9/25/08)
The U.S. budget deficit is projected to total $\$ 1.2$ trillion this year, not $\$ 1.2$ billion.
(Wall Street Journal, 1/13/09)
Total credit limits are $\$ 3.4$ trillion, not billion.
(New York Times, 11/11/09)
The rescue package for ailing European economies is 750 billion euros, not 750 million.
(New York Times, 5/25/10)

In case you were wondering...
"A barrel holds 42 gallons, not 42,000 gallons."
(New York Times, 6/9/10)

Cutting big numbers down to size
"The yearly budget deficit stands at
$\$ 1.3$ billion." (New York Times, 10/24/10)
How much is that per person?
$\$ 1.3$ billion / 300 million is about $\$ 4$ / person
Does that seem a bit low?

The deficit is $\$ 1.3$ trillion, not billion
or about $\$ 4,000 /$ person

Kilo, mega, giga, tera, peta, ...
A petaflop is a thousand trillion instructions per second, not a million trillion.
(New York Times, 3/25/08)

A teravolt or TeV is a million million electron volts, not a billion billion.
(theusdaily.com, 3/30/10)

How many books in the Library of Congress?
WSJ: a book is $2 \times 10^{9} / 1.5 \times 10^{3}$ bytes ~ $10^{6}$ bytes/book

NYT: $10^{21}$ bytes in 100 billion copies
$=10^{21}$ bytes $/ 100 \times 10^{9}$ copies
$=10^{10}$ bytes in 1 copy of all the books
So: $10^{10}$ bytes in 1 copy / $10^{6}$ bytes/book $=10^{4}$ books

The Library of Congress has 10,000 books.
...., exa, zetta, yotta, ...
The Nook e-book reader has 2 GB of memory, "enough to hold about 1,500 digital books."
(Wall Street Journal, 12/9/09)
A zettabyte ( $10^{21}$ bytes) "is equivalent to 100 billion copies of all the books in the Library of Congress."
(New York Times, 12/10/09)
How many books are there in the Library of Congress?

Numeric triage: is the number likely to be much too high, much too low, or plausible?
"Americans receive almost 2 million tons of junk mail daily."

Dear Abby, 1/3/96
2 million tons $=4$ billion pounds 300 million Americans
~ 13 pounds / person / day

Much too high, much too low, or plausible?

## "Each person eats about sixty tons of food in a lifetime."

Microcosm, Carl Zimmer, 2008

## Much too high, much too low, or plausible?

"Shutting down your computer and monitor overnight rather than running them 24 hours a day will save $\$ 88$ a day."
(Newark Star Ledger, 12/08)

Plausibility and common sense

- does the number make sense?
- does it agree with your experience, or run counter to it?
- what would be the implications if the number were accurate?

Faster than a speeding bullet...

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The Times November 16, 2004
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Nasa jet travels 850 miles
in 10 seconds
From Chris Ayres in Los Angeles
AN AIRCRAFT that can fly at ten times the speed of sound will be tested over the Pacific Ocean today - possibly leading to "hypersonic" cruise missiles that could travel from Los Angeles to Pyongyang in less than an hour.

The aircraft was designed by Nasa to travel

## White water?

"The Passaic River was traveling about 200 miles per hour, about five times faster than average." (Newark Star Ledger, 6/10/92)


## Learn to estimate

How many golf balls can fit in a school bus?

http://www.businessinsider.com/15-google-interview-questions-that-will-make-you-feel-stupid-2009-11

## Know some basic facts and conversions

- 6.8 billion people (China 1.3B, India 1.2B, USA 300M, NYC 9M)
- 1 gallon of water weighs $\sim 8 \mathrm{lb}$
- 1 cubic foot of water weighs $\sim 60 \mathrm{lb}$ (rock 200, steel 500)
- 1 liter is just over 1 quart
- 1 kilogram is $\sim 2.2 \mathrm{lb}$ ( 28 gms in an ounce)
- 1 ton is 2000 lb (a metric ton is $1000 \mathrm{~kg} \sim 2200 \mathrm{lb}$ )
- 1 meter is a little over 3 feet or 1 yard
- MP3 music is $1 \mathrm{MB} /$ minute
- electricity costs $15-20$ cents/KWH
- 100K seconds in a day, 500K minutes (30M seconds) in a year
- speed of light: 1 foot/nanosecond, speed of sound: 1000 feet/sec
- 250 working days/year, 2000 hours/year,
- ...

How many petabytes could you fit in this room?


Milestones
"Every day, 10,000 Americans turn
50 years old."
Gambling Magazine, 12/31/05

Little's Law
number in process $=$ arrival rate $\times$ service time

- 300 million Americans (number in process)
- if each lives to 100 (service time)
- 3 million die each year (departure rate)
- 3 million reach any given milestone each year
- if each lives to 75
- 4 million reach any given milestone each year
- about 10,000 reach any milestone each day

Little's Law: a "conservation law"
number in process $=$ arrival rate $\times$ service time

- 6600 Harvard undergrads (number in process)
- each spends 4 years (service time)
- 1650 arrive each year (arrival rate)

1650 leave each year (departure rate)

## Milestones

"Every day, 10,000 Americans turn 50 years old."

Gambling Magazine, 12/31/05
"Every week for the next 18 years, some 88,500 baby boomers will turn 592 ${ }^{2}$."

Newsweek, 9/12/05
"350,000 Americans turn 50 every month."
Forbes, 1/10/05
"4 million students graduate from high school each year."

New York Times, 7/9/10

Some things to watch out for

- errors of dimensionality
- oddly precise numbers
- graphical trickery
- flaky data
- advocacy bias


## Bears in the woods

"Young males can roam 60 to 100 square miles looking for food and mates, but females stay close to the cave, foraging within a 10-mile radius."

Newark Star-Ledger (7/9/99)
How does the area that males cover compare to the more stay-at-home females?

10 mile radius for females:

$$
\begin{aligned}
\text { area } & =\pi r^{2} \\
& =314 \text { square miles! }
\end{aligned}
$$

> New Jersey Real-Time News > Statehouse
N.J. Fish and Game Council approves 6-day black bear hunt
Published: Tuesday, July 13, 2010, 6:13 PM Updated: Wednesday, July 14, 2010, 5:37 AM

> Brian T. Murray/The Star-Ledge

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Story tools

TRENTON - Before the state approved its last bear hunt in 2005, hundreds of farmers, homeowners, animal rights protesters and hunters voiced their opinions in a final, raucous hearing that had to be moved to the State Museum auditorium in Trenton to accommodate the crowd.

The scene was far different today, when the state Fish and Game Council voted unanimously - and with considerably less fanfare - to recommend a six-day bear hunt for northwest New Jersey in December
 Patrick Carr, supervising wildlife biologist, NJ DFW, speaks during a
New Jersey Fish and Game Council meeting in March.


"You know what they say, son -28.35 grams of prevention is worth . 45 kilograms of cure..."

## Specious precision

## 93,754,333 Examples of Data Nonchalance

By TOM ZELLER لt
Published: September 25, 2006
Less than two years into the great cultural awakening to the vulnerability of personal data, companies and institutions of every shape and size - like the data broker ChoicePoint, the credit card processor CardSystems Solutions, media companies like Time Warner and dozens of colleges and universities across the land - have collectively fumbled $93,754,333$ private records. Or at least that's the rough figure the Privacy Rights Clearinghouse, a consumer advocacy organization in San Diego, has tallied thus far.


Darrell Hufif
Illustrated by Irving Geis


Gee-whiz graphs

"Despite the pervasive desire to achieve high grades, students with the initial C or D , presumably because of a fondness for these letters, were slightly less successful at achieving their conscious academic goals than were students with other initials."

GPA by Initial of Name


Gee-whiz graphs (2)


One-dimensional graphs



\$2,000,000
One-dimensional graphs


Between 2000 and 2001, the Graduate School almost quadrupled summer stipends for students in the humanities and the social sciences.

Who's number 1 ?

| National |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Overall score |  |  | tito |  |  |  |  | $\begin{aligned} & \text { \% of } \\ & \text { classes } \\ & \text { clacer } \end{aligned}$ |  |  |  | $t_{i v i t y}$ |  |  |  |  |  | $\begin{gathered} \text { Alumni } \\ \text { oliving } \\ \text { rate } \end{gathered}$ |
|  |  | $\begin{aligned} & \text { Hatio } \\ & \begin{array}{l} \text { are } \\ \text { ight } \end{array} \\ & \hline \end{aligned}$ |  | reshman retention rate |  | 2000 rad rate |  | $\begin{aligned} & \text { acaulty } \\ & \text { sourd } \\ & \text { rank } \end{aligned}$ |  | $\begin{aligned} & \text { \% with } \\ & 50 \text { or } \\ & \text { more } \end{aligned}$ |  | $\begin{aligned} & \% \text { of } \\ & \text { fullime } \end{aligned}$ faculty |  | SAT/ACT ercentile |  | tance |  |  |  |
|  |  |  |  |  |  |  | ort b |  | Rank |  |  |  |  |  |  |  |  |  |  |
| 1. Princeton University | 100 | 4.9 |  | 1 99\% | 92\% | \% 97\% | $+5$ |  | 71\% | 11\% | 6/1 | 92\% |  | 1350. 1540 | 92\% | 12\% |  |  |  |
| 2. Hancard University | 99.0 | 4.9 | 2 | 96\% | 91\% | \% 97\% | +6 |  | 69\% | 13\% | 8/1 | 91\% | 1 | $\begin{aligned} & 1410 \\ & 1580 \end{aligned}$ | 90\% | 11\% |  |  | (46\% |
| Yale University (CT) | 99.0 | 4.9 | 3 | 3 98\% | \% | \% 95 | +2 | 10 | 75 | 8\% | 7/1 | 90\% | 2 | $\begin{aligned} & 1380 \\ & 1550 \end{aligned}$ | 95\% | 16\% |  |  |  |
| 4. California Institute | 6.0 | 4.7 | 25 | 94\% | 95\% | 82\% | 13 | 2 | 76\% | $5 \%$ | 3/ | 968 | 5 | $\begin{aligned} & 1450 \\ & 1580 \end{aligned}$ | 98 | 13\% |  | 113 | 39\% |
| 5. $\qquad$ Massachusetts Inst | 95.0 | 4.9 | 10 | 98\% | 3\% | 91\% | . 2 | 19 | 58\% | 16\% | 7/1 | 89\% | 3 | $\begin{gathered} 140 \\ 1560 \end{gathered}$ | 97\% | 16\% |  |  |  |
| $\begin{aligned} & \text { Stanford University } \\ & \text { (CA) } \end{aligned}$ | 5.0 | 4.9 | 7 | 98\% | 91\% | \% 93\% | +2 | 15 | 69\% | 14\% | $7 / 1$ | 998 | 6 | $\begin{aligned} & 1360 \\ & 1560 \end{aligned}$ | 89\% | 138 |  | 911 | 140 |
| University of Pennsrluania | 95.0 | 4.5 | 15 | 97\% | 87\% | 91* | +4 |  | 70 | 8\% | 7/1 | 89\% | 8 | $\begin{aligned} & 1310 \\ & 1490 \end{aligned}$ | 92\% | 23\% |  | 510 |  |
| 8. Duke University (NC) | 93.0 | 4.6 |  | 97\% | 91\% | 93\% | +2 |  | 69\% | 7* | 9/1 | 98\% | 12 | $\begin{aligned} & 1300- \\ & 1500 \\ & \hline \end{aligned}$ | 86\% | 26 | 10 |  | 45\% |
| 9. $\frac{\text { Columbia University }}{\text { (Nr) }}$ | ( 89.0 | 4.8 | 14 | 98\% | 36\% | 88\% | +2 | 14 | 88\% | 9\% | 7/1 | 93 | 7 | $\begin{aligned} & 130 \\ & 1510 \end{aligned}$ | 87\% | 13 |  | 21 | 32\% |
| $\begin{aligned} & \text { Dartmouth College } \\ & \text { (NH) } \end{aligned}$ | 89.0 | 4.4 |  | 96\% | 91\% | 92\% | +1 | 20 | 57\% | 118 | $9 / 1$ | 88\% | 10 | $\begin{aligned} & 1330- \\ & 1520 \\ & \hline \end{aligned}$ | 86\% | 21\% | 1 |  | 48\% |
| University of Chioago | 89.0 | 4.7 | 24 | 94\% | 86\% | 85\% | -1 |  | 68\% | 3\% | 4/1 | 95\% | 22 | $\begin{aligned} & 1280- \\ & 1490 \end{aligned}$ | 82\% | 44 | 15 | 520 | 33\% |

Much too high, much too low, or plausible?
"Each year, according to the [American Anorexia and Bulimia] Association, 150,000 American women die of anorexia."

Naomi Wolf, The Beauty Myth, 1990, p 148

Who says so? How do they know?
Why do they care? What are they selling?
"Four thousand teens will try their first cigarette today."
advertisement, New York Times (11/18/05)
"Endorsed by American Academy of Pediatrics, American Heart Assoc, American Lung Assoc, American Medical Assoc, National PTA."
"Every day 5000 teenagers try pot for the first time."
advertisement, New York Times (11/4/05)
Coalition for a Drug-Free America

Much too high, much too low, or plausible?
"The number of American children killed by guns has doubled every year since 1950."

Nancy Day, Violence in Schools: Learning in Fear (1996)

## Defenses

- recognize the enemy
- flaky numbers / excessive precision
- arithmetic errors/ wrong units / wrong dimensions
- bad statistics / deceptive presentation / suspect motives
- beware of the source
- how do they know? how could they know?
- why do they care? what are they trying to sell you?
- learn some useful numbers, facts, shortcuts
- populations, rates, sizes, areas, conversions, etc.
- approximate arithmetic, Little's Law, Rule of 72 , powers of 2
- use your common sense and experience
- much too big? much too small?
- does it makes sense? what would it imply?

Good reading

- Darrell Huff, How to Lie with Statistics
- Joel Best, Damned Lies and Statistics, More Damned Lies and Statistics, ...
- John Allen Paulos, Innumeracy,

A Mathematician Reads the Newspaper

- Web sites
innumeracy.com
megapenny.com
math.temple.edu/~paulos/

