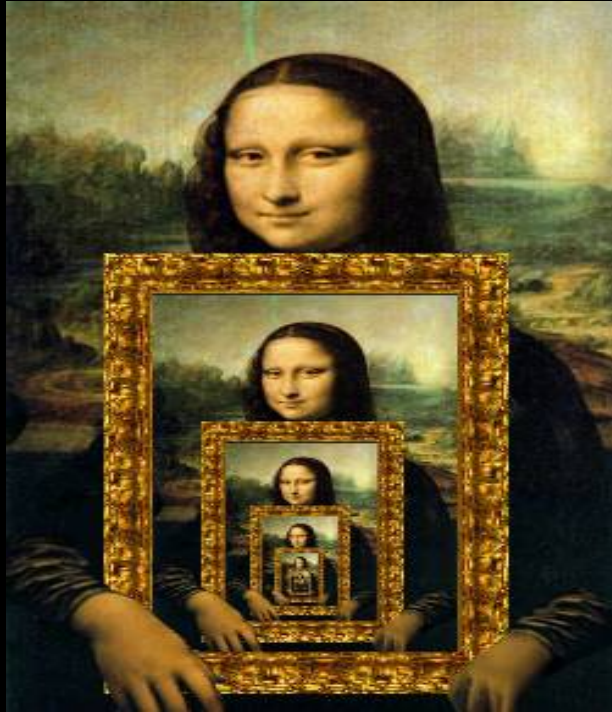


Recursion



Adding all numbers from 1 to n

$$\text{sum}(n) = n + (n - 1) + (n - 2) + \dots + 1$$

Adding all numbers from 1 to n

$$\text{sum}(n) = n + (n - 1) + (n - 2) + \dots + 1$$

$$\text{sum}(n) = n + \text{sum}(n - 1)$$

Adding all numbers from 1 to n

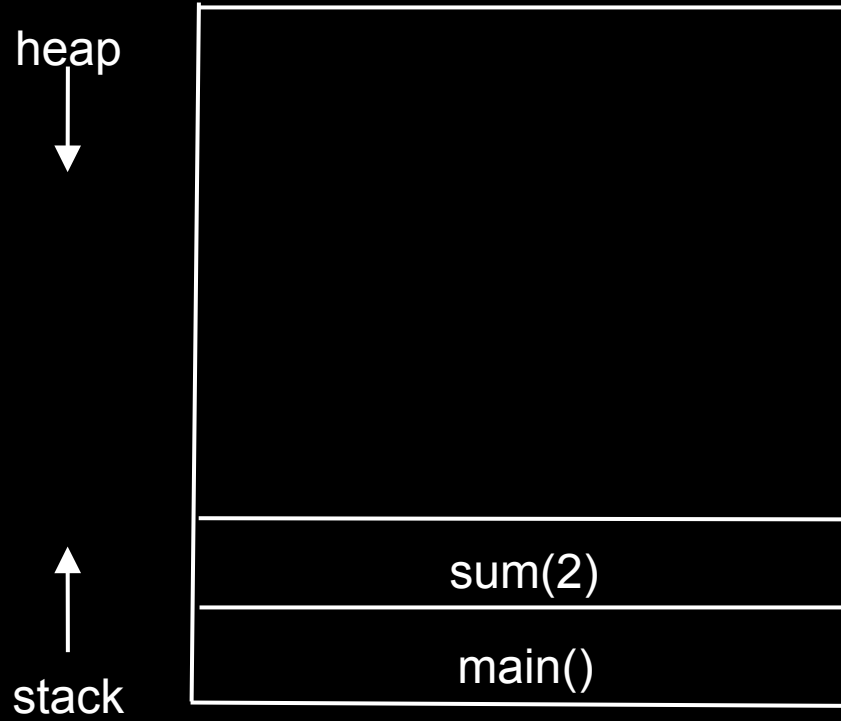
$$\text{sum}(n) = n + \mathbf{(n - 1)} + \mathbf{(n - 2)} + \dots + \mathbf{1}$$

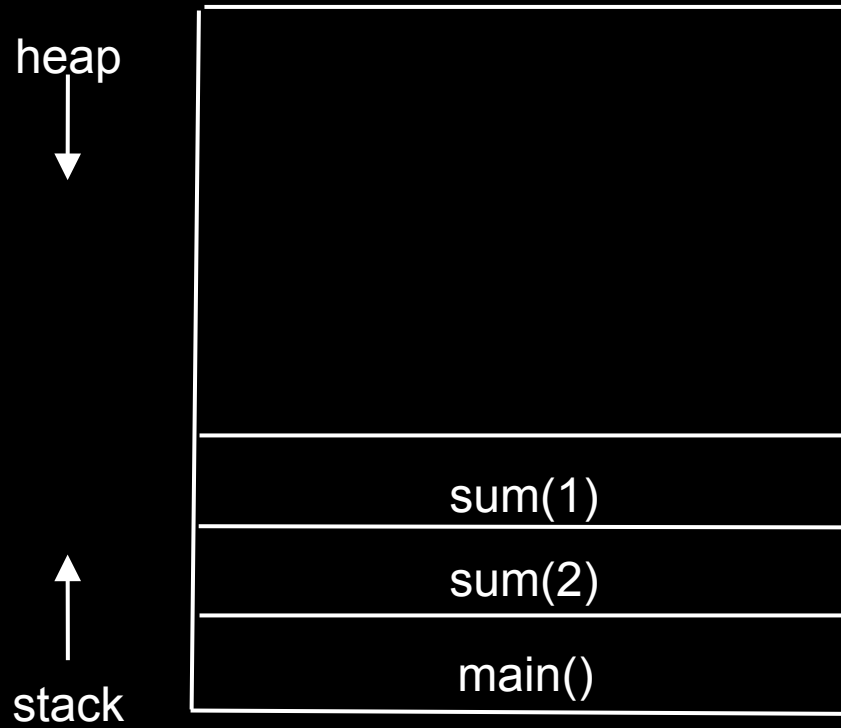
$$\text{sum}(n) = n + \mathbf{\text{sum}(n - 1)}$$

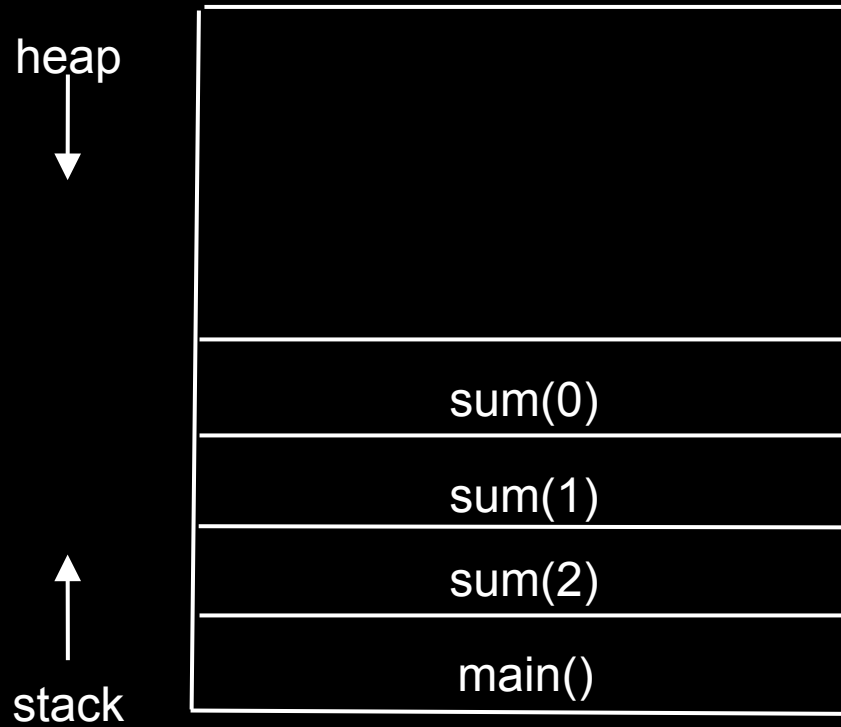
$$\text{sum}(0) = 0 \text{ (base case)}$$

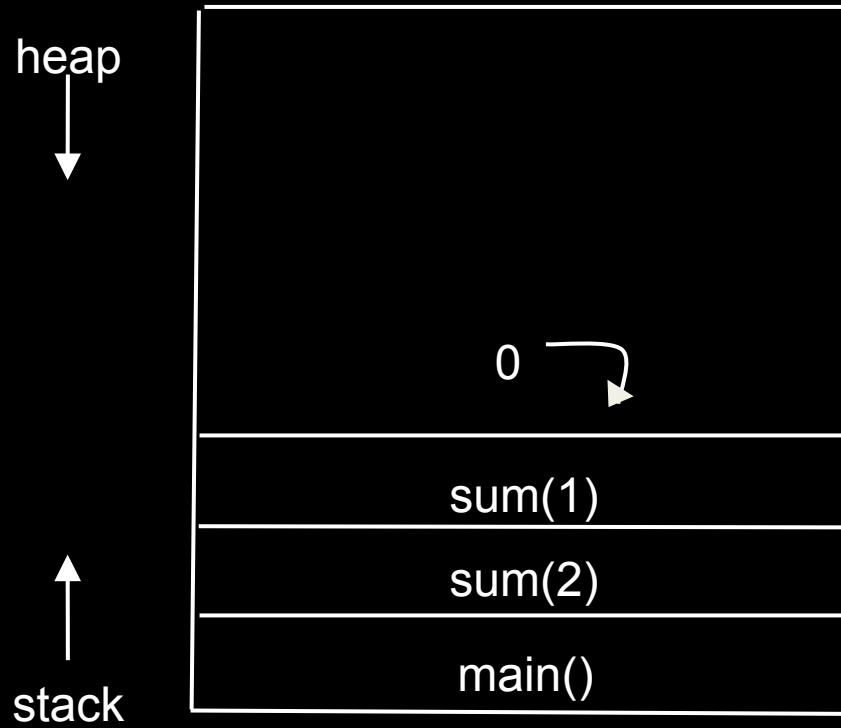
```
int sum(int n)
{
    if (n <= 0)
    {
        return 0;
    }
    else
    {
        return n + sum(n - 1);
    }
}
```

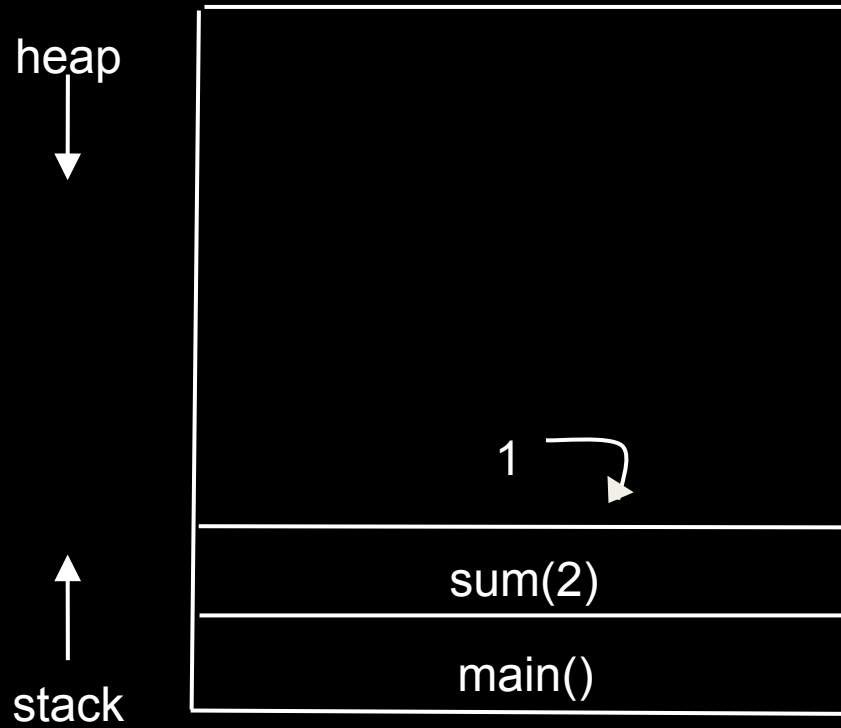
$$\begin{aligned} \text{sum}(2) &= 2 + \text{sum}(1) \\ &\quad \underbrace{\hspace{10em}} \\ &\quad 1 + \text{sum}(0) \\ &\quad \quad \underbrace{\hspace{4em}} \\ &\quad \quad 0 \end{aligned}$$

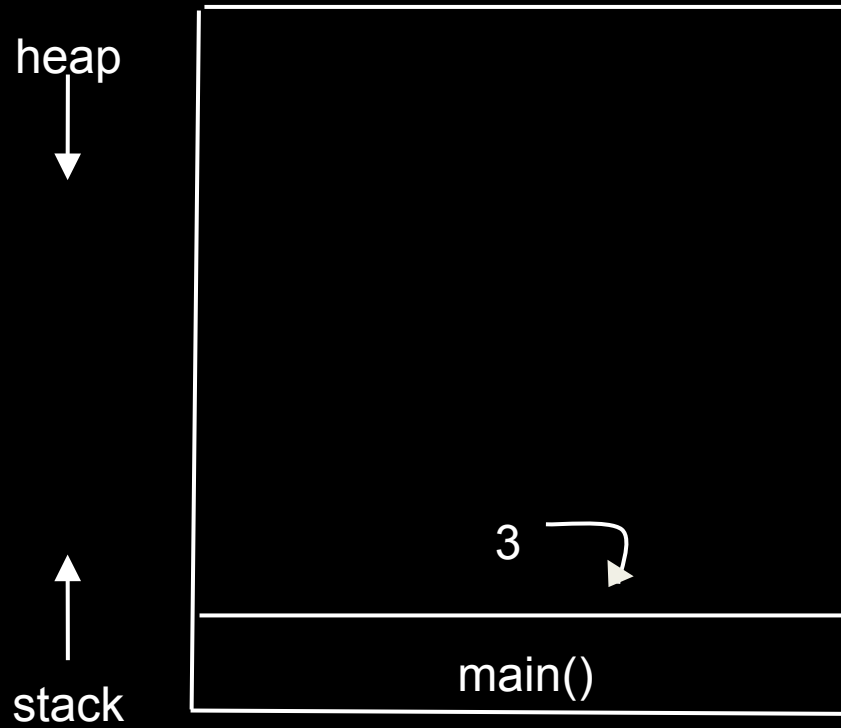












Sorting

- Dictionary
- Facebook
- Organizing data
- Pokémon!

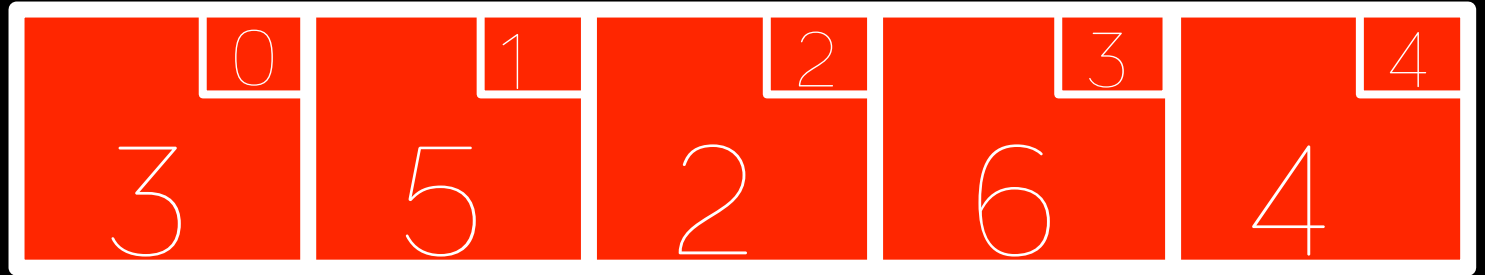
Selection Sort

1. **Select** the smallest unsorted value
2. Move that value to the end of the “sorted” part of the list
3. Repeat from step 1 if there are still unsorted items

All values start as **Unsorted**

Sorted

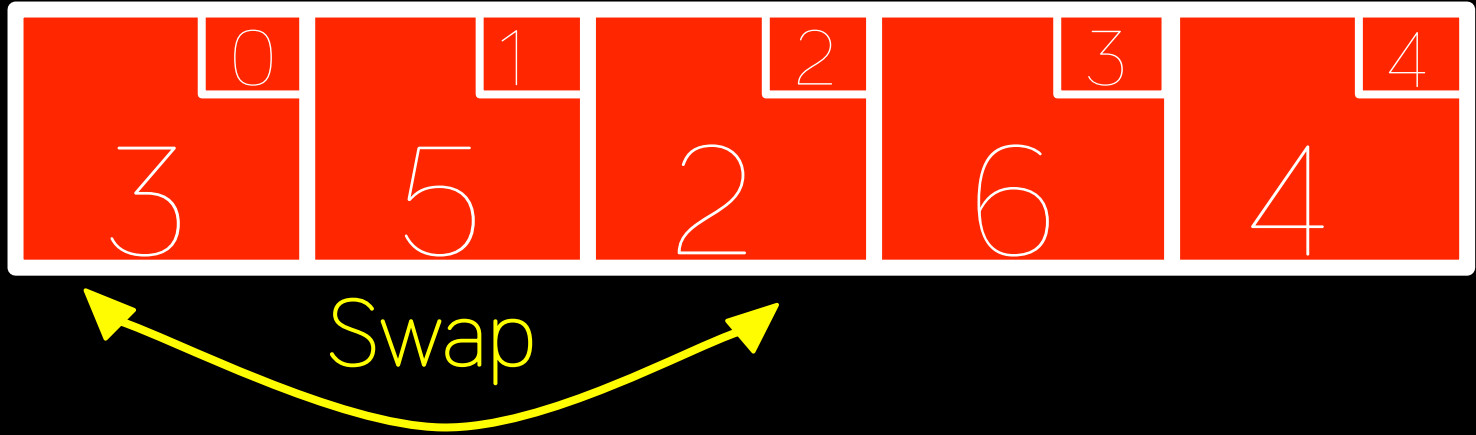
Unsorted



First pass:
2 is smallest, swap with 3

Sorted

Unsorted

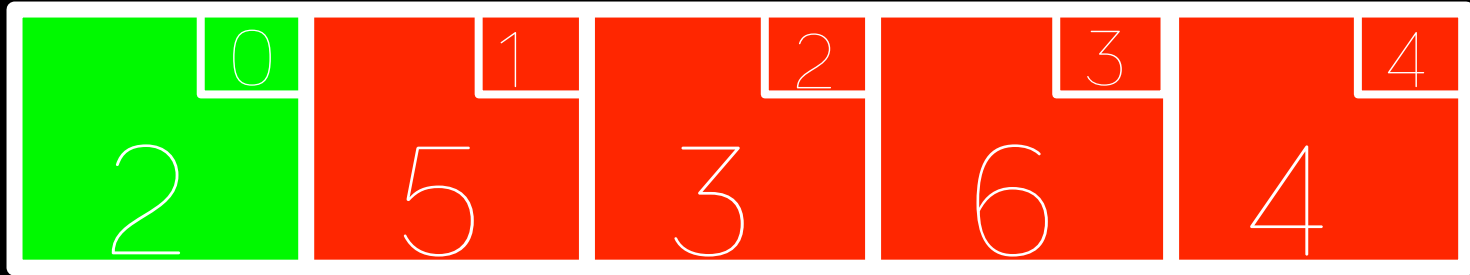


Second pass:

3 is smallest, swap with 5

Sorted

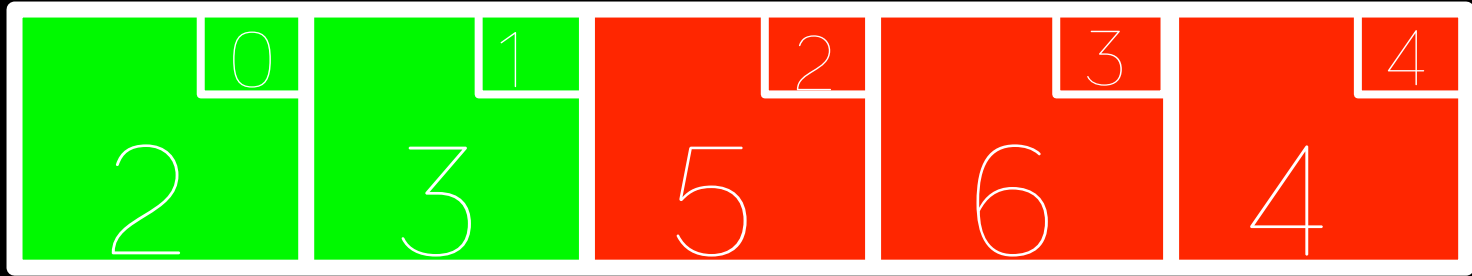
Unsorted



Third pass:
4 is smallest, swap with 5

Sorted

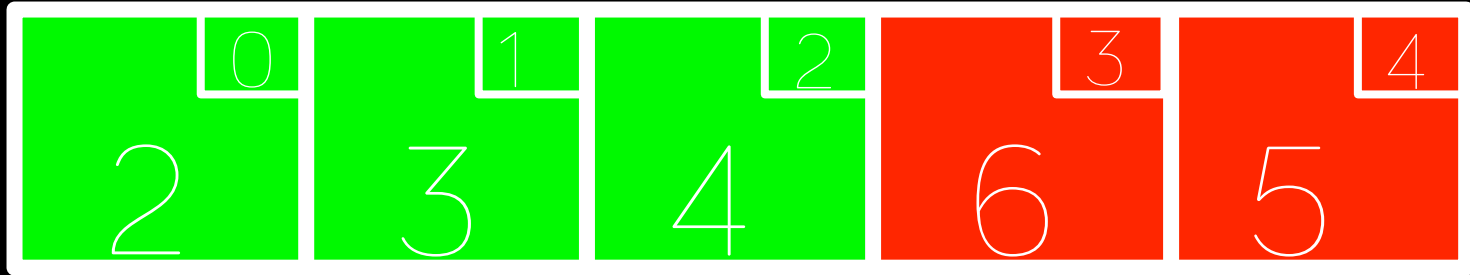
Unsorted



Fourth pass:
5 is smallest, swap with 6

Sorted

Unsorted

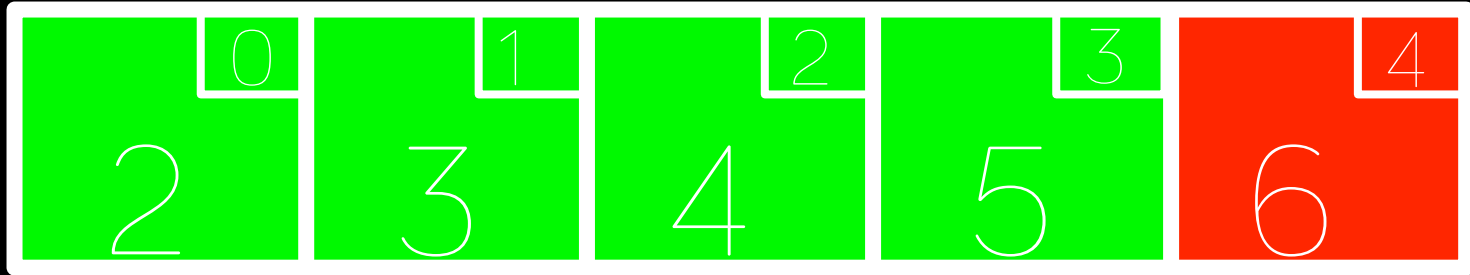


Swap

Fifth pass:
6 is the only value left, done!

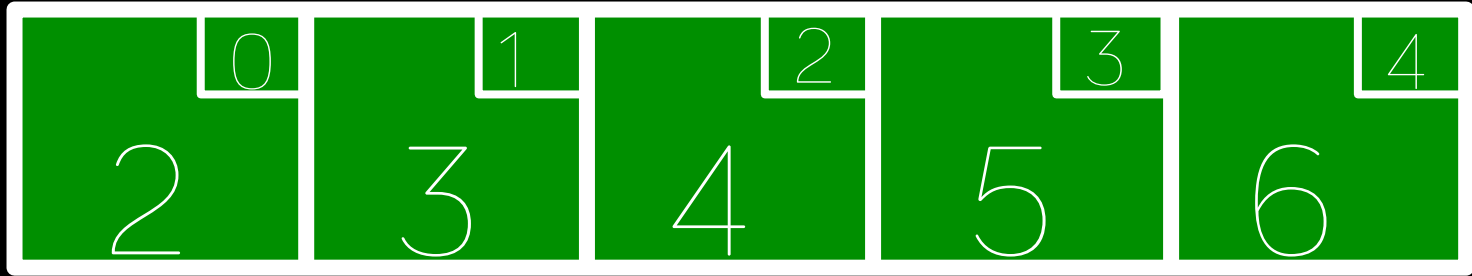
Sorted

Unsorted



Fifth pass:
6 is the only value left, done!

All Sorted



Insertion Sort

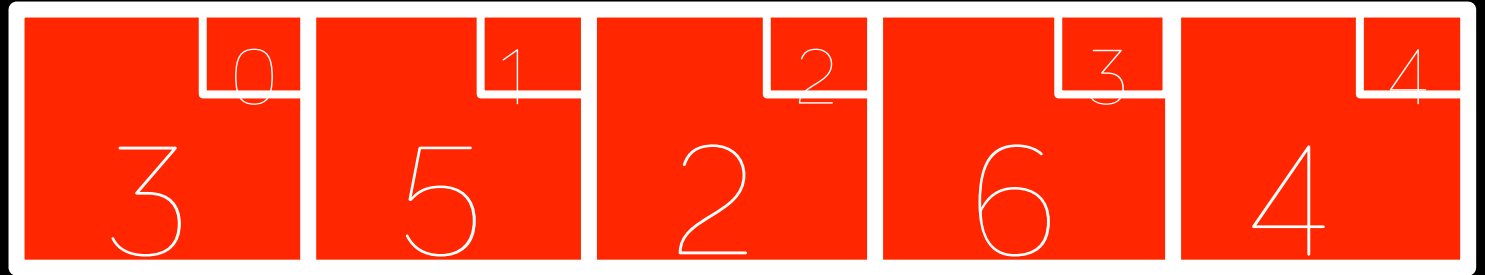
For each unsorted element n :

1. Determine where to insert n on the sorted portion of the list
2. Shift sorted elements rightwards as necessary to make room for n
3. **Insert** n into sorted portion of the list

All values start as **Unsorted**

Sorted

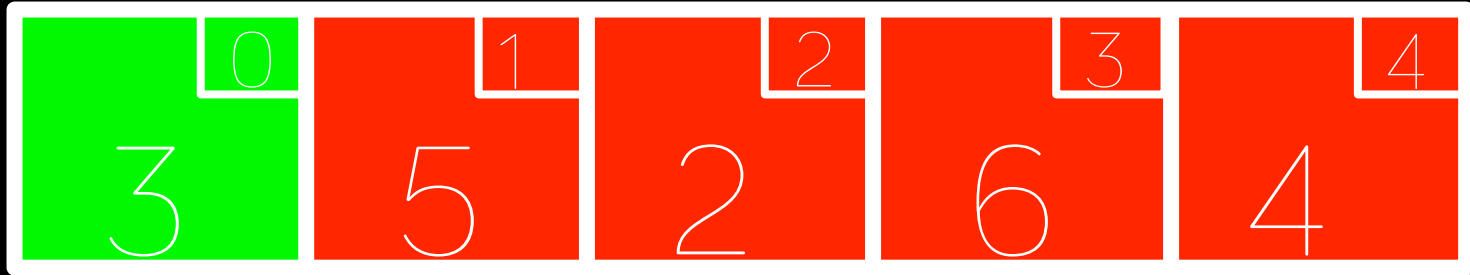
Unsorted



Add first value to **Sorted**

Sorted

Unsorted

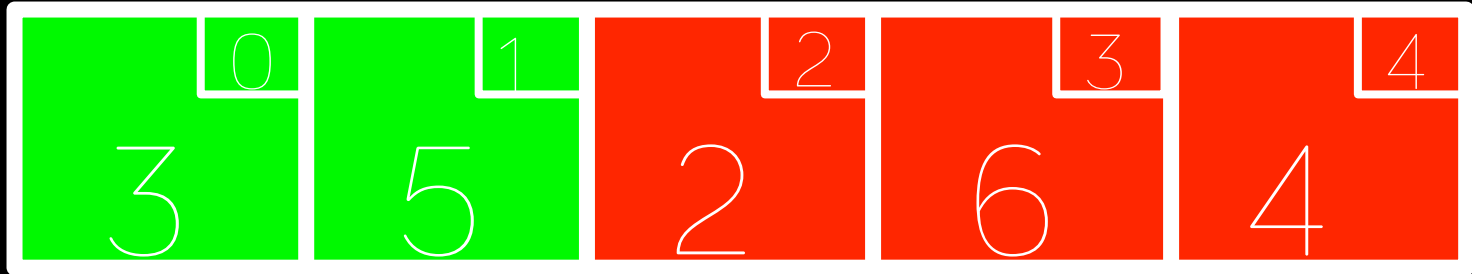


$$5 > 3$$

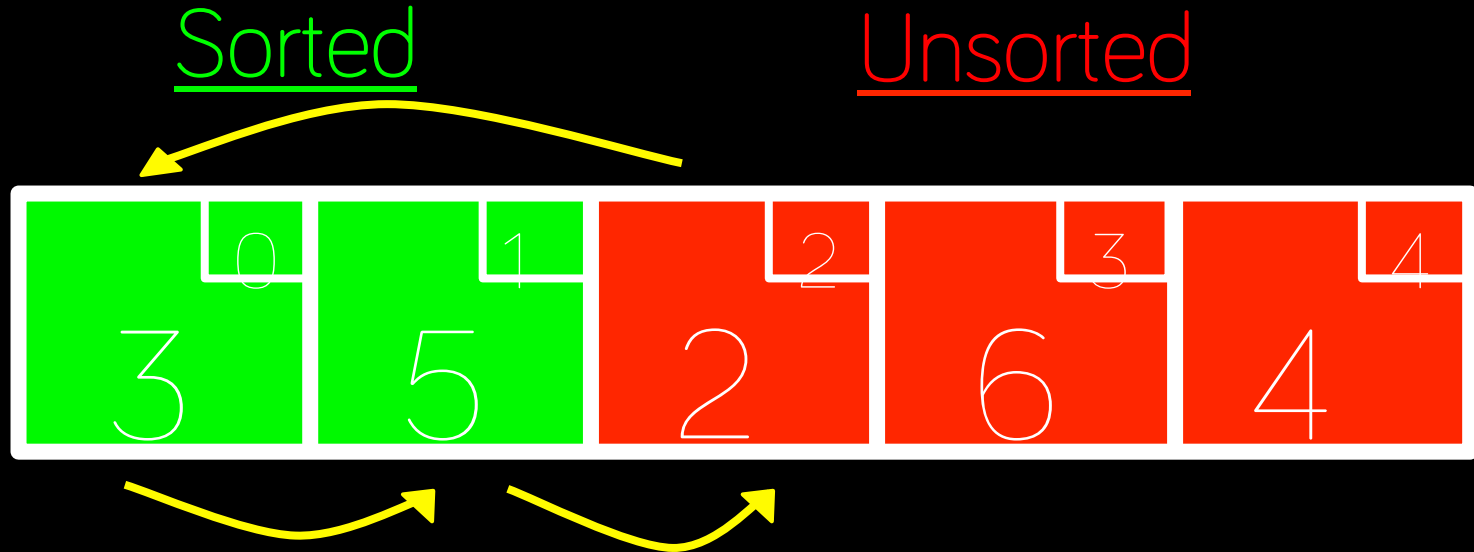
insert 5 to right of 3

Sorted

Unsorted



$2 < 5$ and $2 < 3$
shift 3 and 5
insert 2 to left of 3

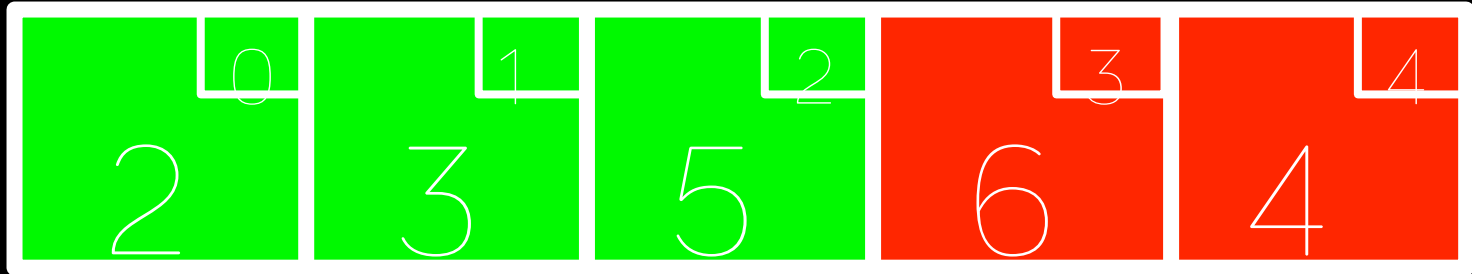


$$6 > 5$$

insert 6 to right of 5

Sorted

Unsorted



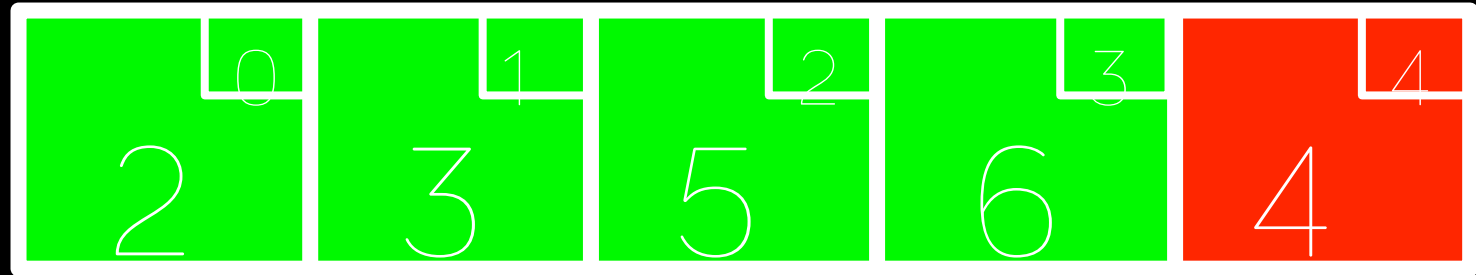
$4 < 6$, $4 < 5$, and $4 > 3$

shift 5 and 6

insert 4 to right of 3

Sorted

Unsorted



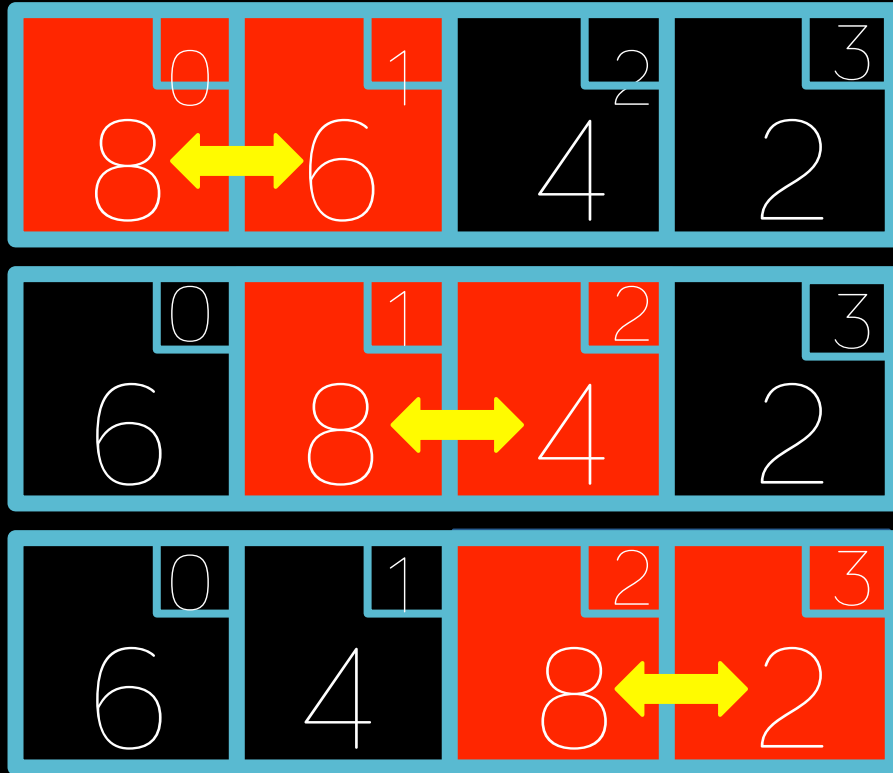
Bubble Sort

1. Step through entire list, swapping adjacent values if not in order
2. Repeat from step 1 if any swaps have been made

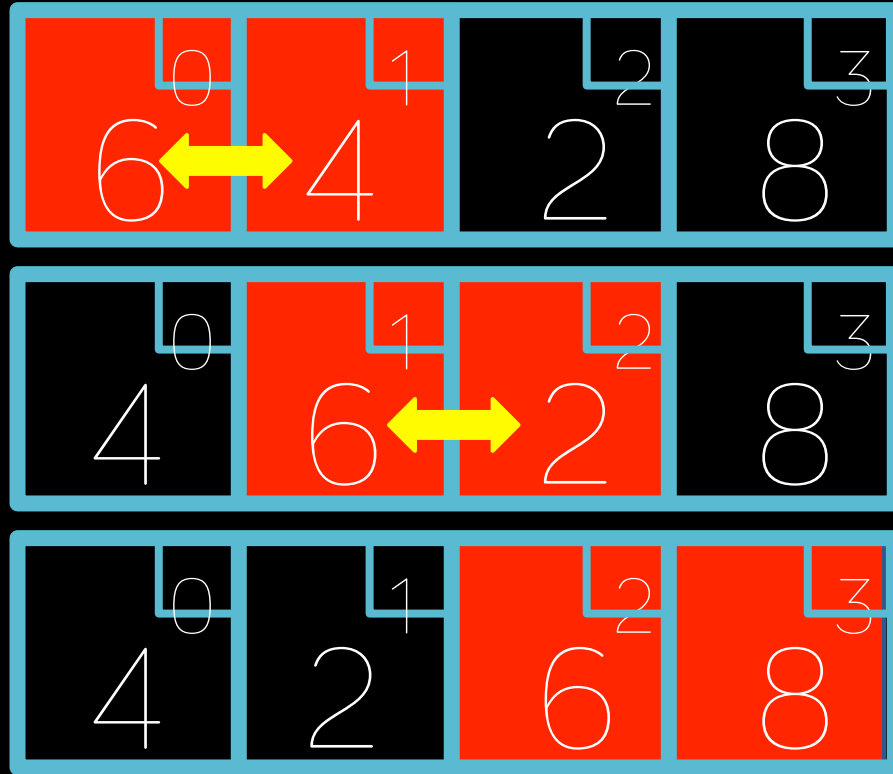
At each step, the largest value **bubbles** to the end of the list

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | | 1 | | 2 | | 3 |
| 8 | | 6 | | 4 | | 2 | |

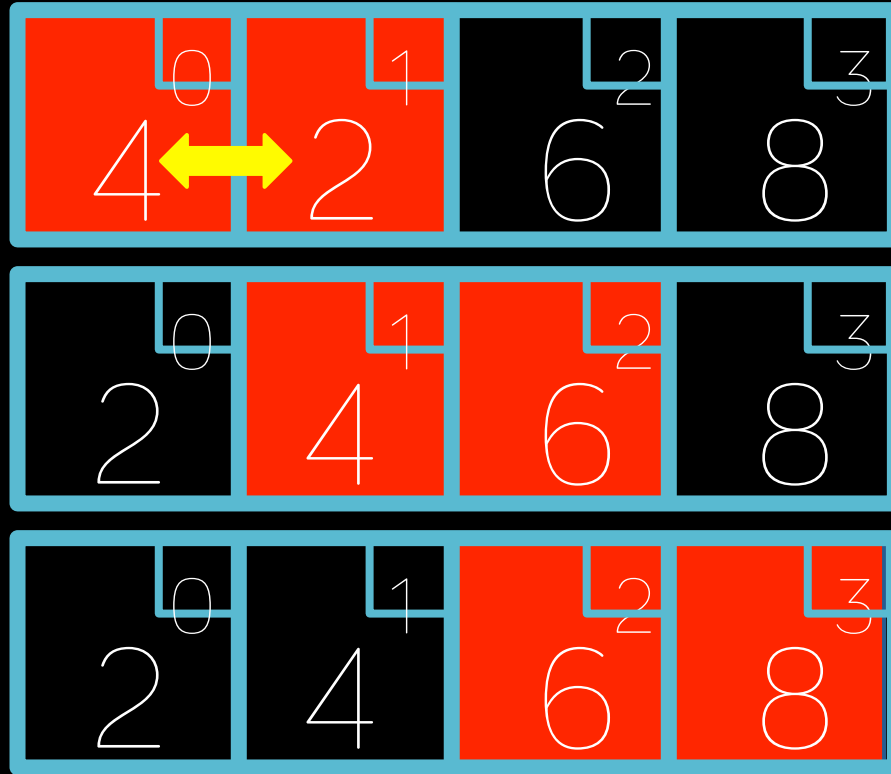
First pass: 3 swaps



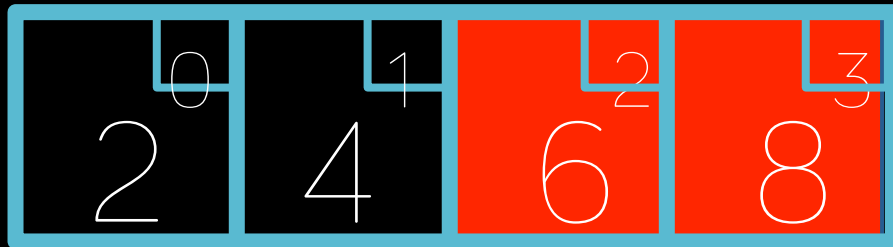
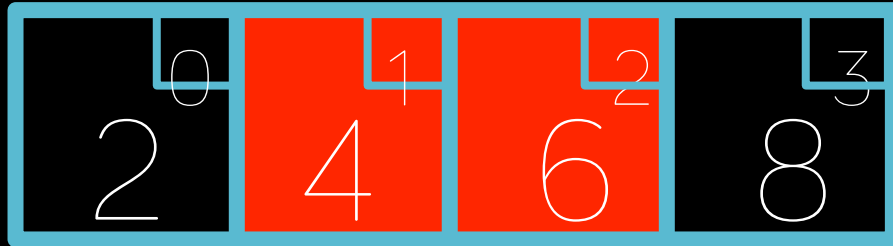
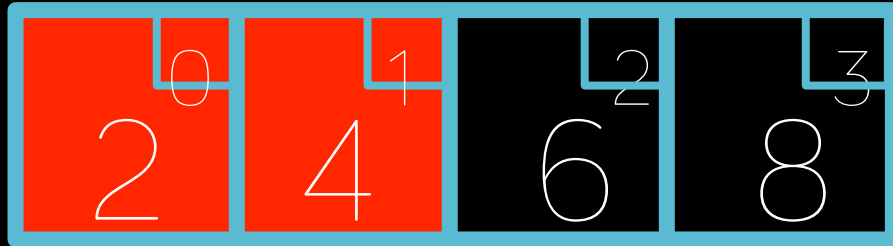
Second pass: 2 swaps



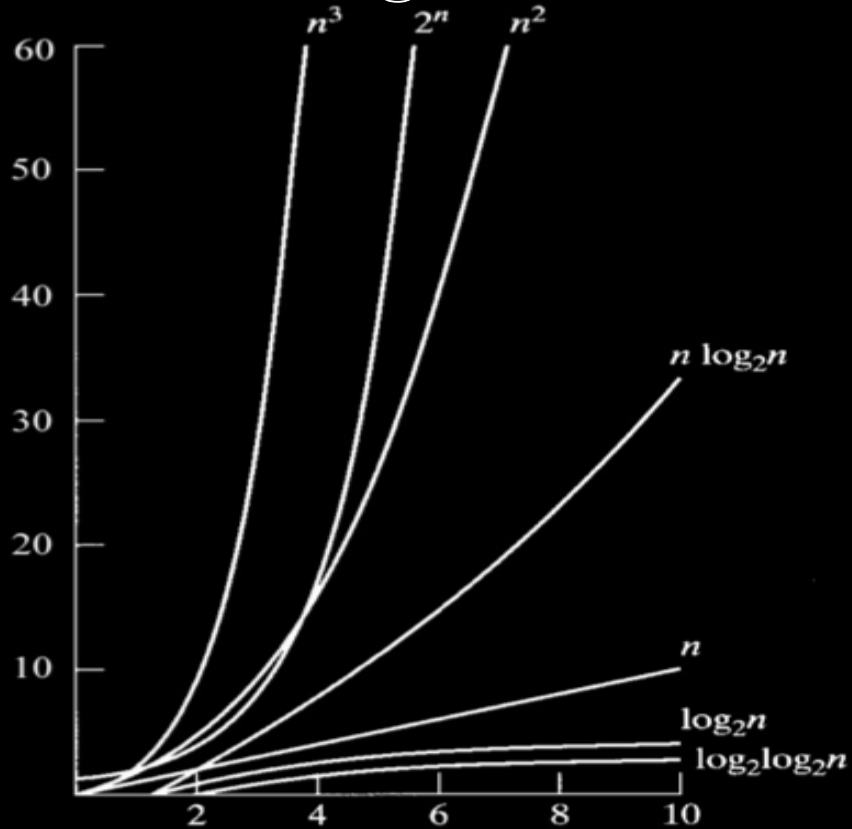
Third pass: 1 swap



Fourth pass: 0 swaps



Running Time

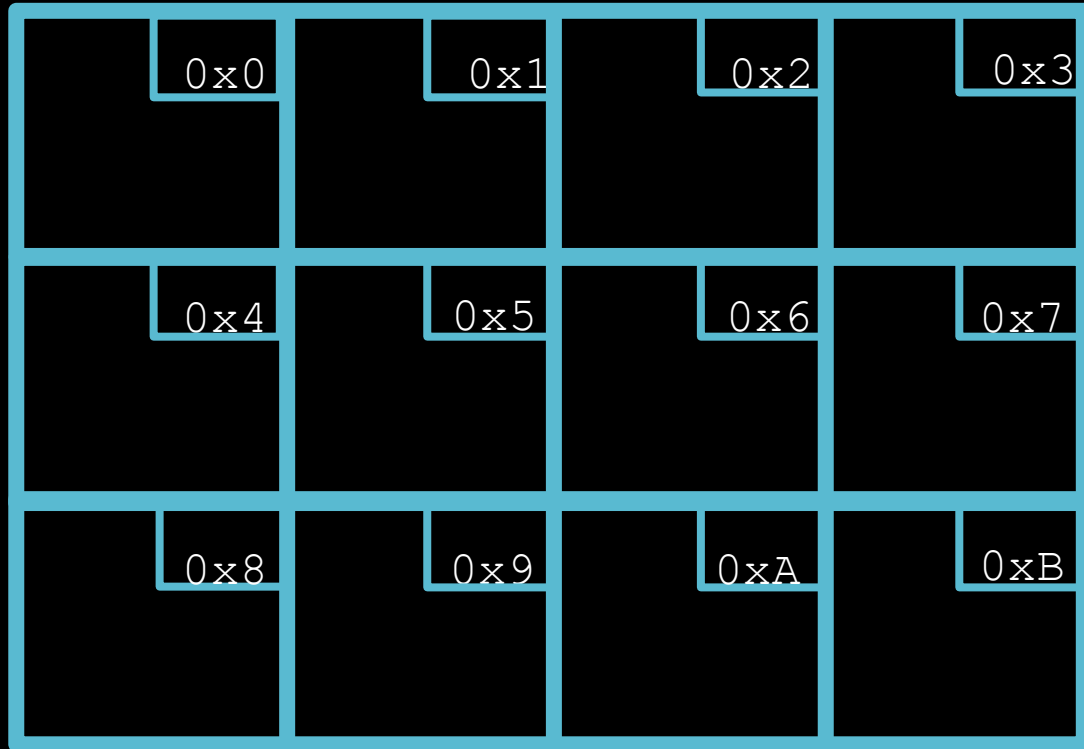


| | O | Ω | Θ |
|----------------|------------|------------|------------|
| Bubble sort | n^2 | n | |
| Selection sort | n^2 | n^2 | n^2 |
| Insertion sort | n^2 | n | |
| Merge sort | $n \log n$ | $n \log n$ | $n \log n$ |

Pointers



Memory



Creating Pointers

Declaring pointers:

<type>* <variable name>

Examples:

```
int* x;
```

```
char* y;
```

```
float* z;
```

Size: 4 bytes for 32-bit machine

Referencing and Dereferencing

Referencing:

&<variable name>

& is the same as saying “address of”

Dereferencing:

*****<pointer name>

* is the same as saying “content of”

Let's see this in memory

```
int a = 3;
```

```
int b = 4;
```

```
int c = 5;
```

```
int* pa = &a;
```

```
int* pb = &b;
```

```
int* pc = &c;
```

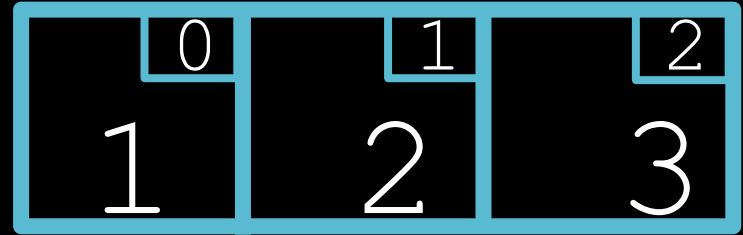
Pointers and Arrays

```
int array[3];
```

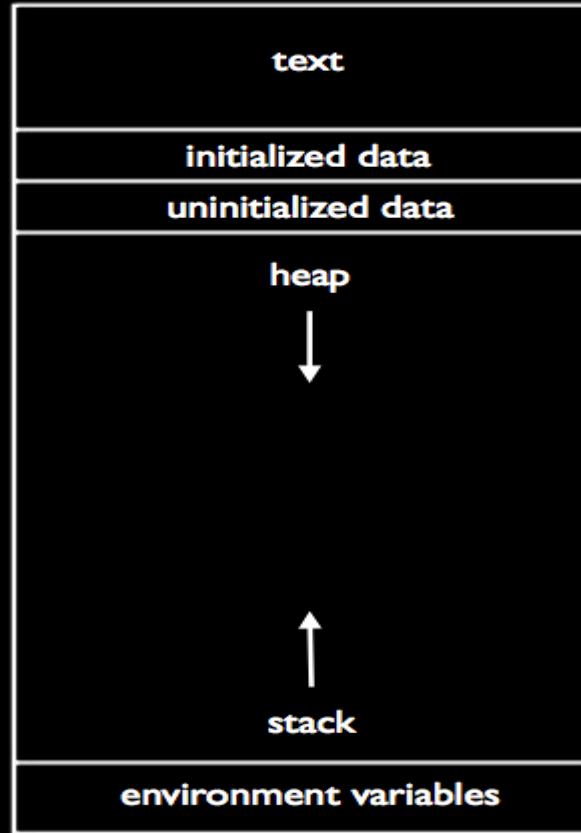
```
*array = 1;
```

```
*(array + 1) = 2;
```

```
*(array + 2) = 3;
```



Dynamic Memory Allocation



m(emory)alloc(ation)

- Allocates memory on the heap
- Returns pointer to allocated memory
- Returns **NULL** if can't allocate
memory

malloc
`void* malloc(size in bytes);`

example:

```
int* ptr;
```

```
ptr = malloc(sizeof(int) * 10);
```

Check for NULL!

```
int* ptr = malloc(sizeof(int) * 10);  
  
if (ptr == NULL)  
{  
    printf("Error -- out of memory.\n");  
    return 1;  
}
```

Don't forget to free!

```
free(ptr);
```

```
#include <cs50.h>
#include <stdio.h>

int main(void)
{
    int* ptr = malloc(sizeof(int));
    if (ptr == NULL)
    {
        printf("Error -- out of memory.\n");
        return 1;
    }

    *ptr = GetInt();
    printf("You entered %i.\n", *ptr);

    free(ptr);

    return 0;
}
```