- We might describe an implementation of an algorithm as being particularly "elegant" if it solves a problem in a way that is both interesting and easy to visualize.
- The technique of **recursion** is a very common way to implement such an "elegant" solution.

• The definition of a recursive function is one that, as part of its execution, invokes itself.

- The factorial function (*n*!) is defined over all positive integers.
- n! equals all of the positive integers less than or equal to n, multiplied together.

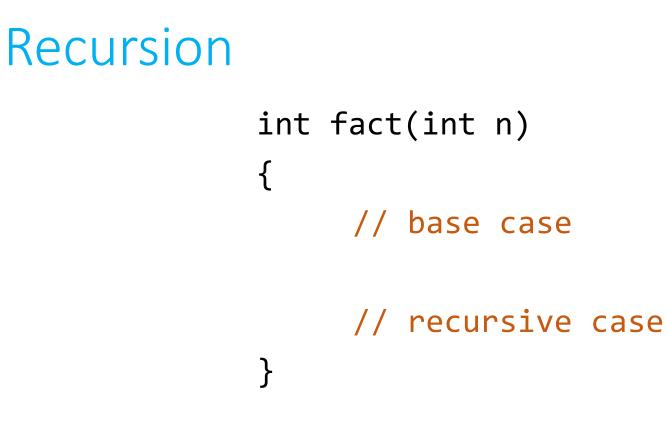
 Thinking in terms of programming, we'll define the mathematical function n! as fact(n).

| <pre>fact(1)</pre> | = | 1 | | | | | | | | |
|--------------------|---|---|---|----|-----|-----|----|---|---|---|
| fact(2) | = | 2 | * | fa | act | t(1 | L) | | | |
| <pre>fact(3)</pre> | = | 3 | * | 2 | * | 1 | | | | |
| <pre>fact(4)</pre> | = | 4 | * | 3 | * | 2 | * | 1 | | |
| fact(5) | = | 5 | * | 4 | * | 3 | * | 2 | * | 1 |

fact(n) = n * fact(n-1)

• This forms the basis for a **recursive definition** of the factorial function.

- Every recursive function has two cases that could apply, given any input.
 - The *base case*, which when triggered will terminate the recursive process.
 - The *recursive case*, which is where the recursion will actually occur.



```
Recursion
              int fact(int n)
              {
                   if (n == 1)
                   {
                         return 1;
                   }
                   // recursive case
              }
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                         return n * fact(n-1);
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• In general, but not always, recursive functions replace loops in non-recursive functions.

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```
int fact(int n)
{
    if (n == 1)
        return 1;
    else
        return n * fact(n-1);
}
```

```
int fact2(int n)
    int product = 1;
    while(n > 0)
        product *= n;
        n--;
    return product;
```

• In general, but not always, recursive functions replace loops in non-recursive functions.

• It's also possible to have more than one base or recursive case, if the program might recurse or terminate in different ways, depending on the input being passed in.

- Multiple base cases: The Fibonacci number sequence is defined as follows:
 - The first element is 0.
 - The second element is 1.
 - The n^{th} element is the sum of the $(n-1)^{\text{th}}$ and $(n-2)^{\text{th}}$ elements.
- Multiple recursive cases: The Collatz conjecture.

- The Collatz conjecture is applies to positive integers and speculates that it is always possible to get "back to 1" if you follow these steps:
 - If *n* is 1, stop.
 - Otherwise, if *n* is even, repeat this process on *n*/2.
 - Otherwise, if *n* is odd, repeat this process on 3n + 1.
- Write a recursive function collatz(n) that calculates how many steps it takes to get to 1 if you start from n and recurse as indicated above.

| n | collatz(n) | Steps |
|----|------------|--|
| 1 | 0 | 1 |
| 2 | 1 | $2 \rightarrow 1$ |
| 3 | 7 | $3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 4 | 2 | $4 \rightarrow 2 \rightarrow 1$ |
| 5 | 5 | $5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 6 | 8 | $6 \rightarrow 3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 7 | 16 | $7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 8 | 3 | $8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 15 | 17 | $15 \rightarrow 46 \rightarrow 23 \rightarrow 70 \rightarrow \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 27 | 111 | $27 \rightarrow 82 \rightarrow 41 \rightarrow 124 \rightarrow \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |
| 50 | 24 | $50 \rightarrow 25 \rightarrow 76 \rightarrow 38 \rightarrow \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ |

```
Recursion
```

```
int collatz(int n)
{
    // base case
    if (n == 1)
        return 0;
    // even numbers
    else if ((n % 2) == 0)
        return 1 + collatz(n/2);
    // odd numbers
    else
        return 1 + collatz(3*n + 1);
}
```