Computer Science 50

Introduction to Computer Science I

Harvard College

Week 3

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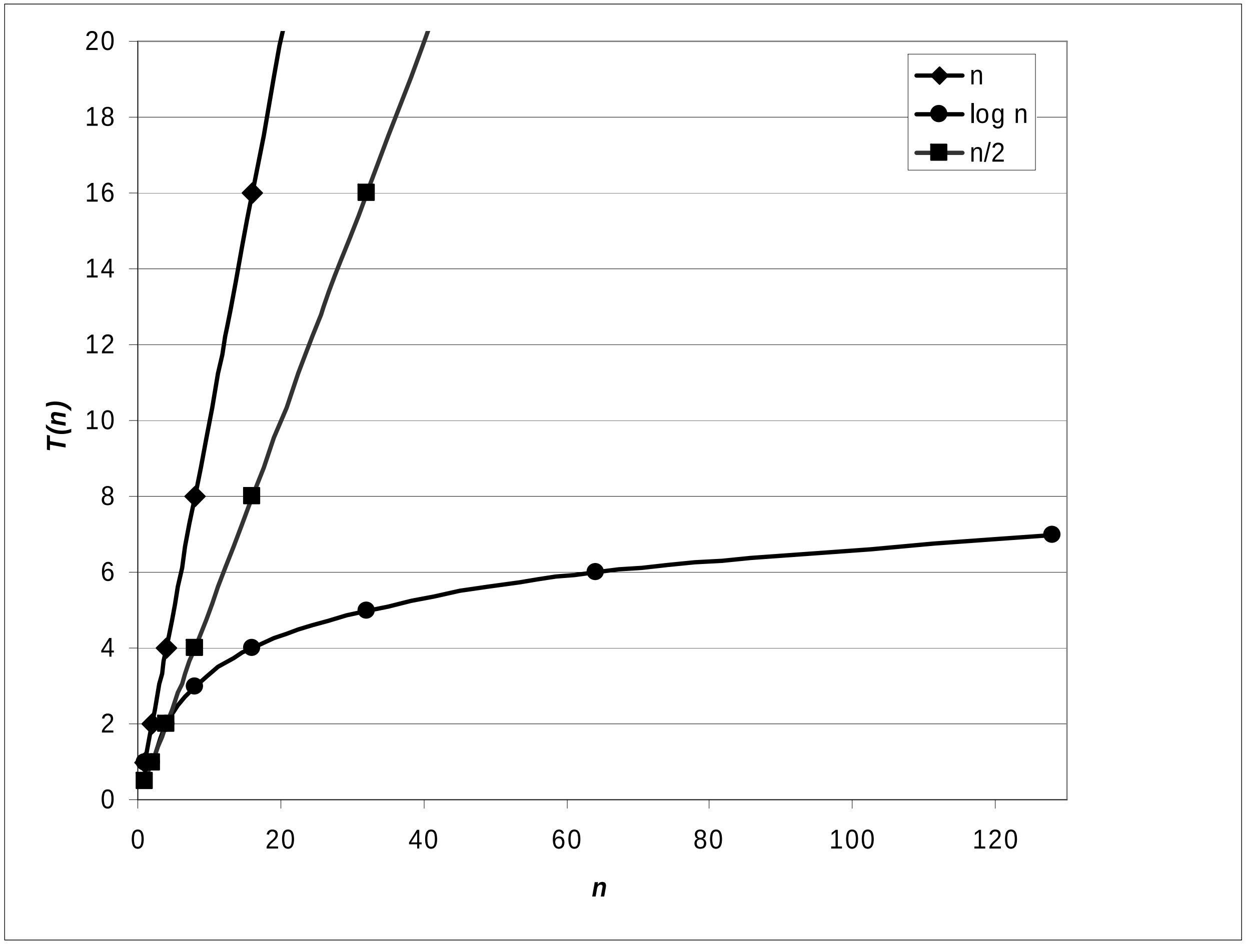
Divide and Conquer

Parallel Processing

1) Stand up.
2) Assign yourself the number 1.
3) Find someone else that is standing up.
   (If no one is standing, remain standing until I call on you.)
4) Add your number to that person’s number;
   the total is your new number.
5) One of you should then sit down.
6) If you’re still standing, go back to step 3.

Running Time
T(n)



Running Time

T(*n*)

$\log_2\log_2 n$	$\log_2 n$	n	$n \log_2 n$	n^2	n^3	2^n
—	0	1	0	1	1	2
0	1	2	2	4	8	4
1	2	4	8	16	64	16
1.58	3	8	24	64	512	256
2	4	16	64	256	4096	65536
2.32	5	32	160	1024	32768	4294967296
2.6	6	64	384	4096	2.6×10^5	1.85×10^{19}
3	8	256	2.05×10^3	6.55×10^4	1.68×10^7	1.16×10^{77}
3.32	10	1024	1.02×10^4	1.05×10^6	1.07×10^9	1.8×10^{308}
4.32	20	1048576	2.1×10^7	1.1×10^{12}	1.15×10^{18}	6.7×10^{315652}

TABLE 7.1 COMMON COMPUTING TIME FUNCTIONS

Figure from *C++: An Introduction to Data Structures*, by Larry Nyhoff.

Asymptotic Notation

Informally

$$O$$

$$\Theta$$

$$\Omega$$

Asymptotic Notation

Formally

$T(n) \in O(f(n))$

We say that the running time, $T(n)$, of an algorithm is "in big O of f of n" iff there exist an integer $n_0 > 0$ and a real number $c > 0$ such that $T(n) \leq c \cdot f(n)$ for all $n \geq n_0$.

$T(n) \in \Theta(f(n))$

We say that the running time, $T(n)$, of an algorithm is "in theta of f of n" iff there exist an integer n_0 and real numbers $c_1, c_2 > 0$ such that $c_1 \cdot f(n) \leq T(n) \leq c_2 \cdot f(n)$ for all $n \geq n_0$.

$T(n) \in \Omega(f(n))$

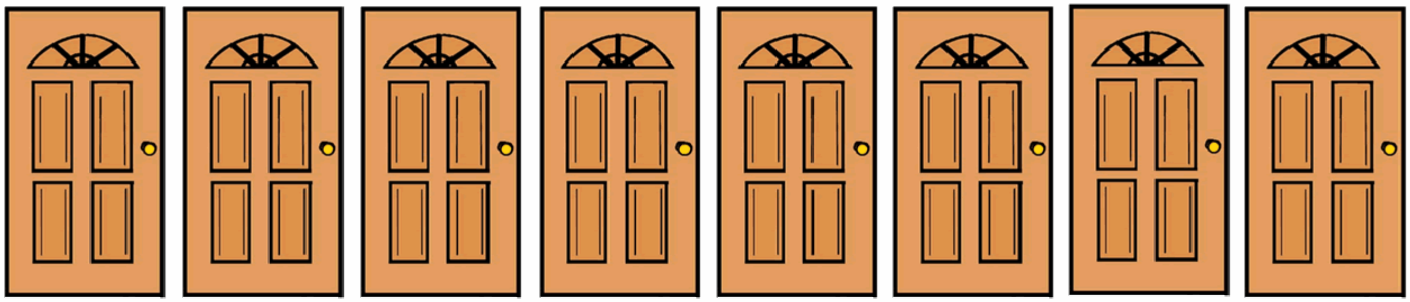
We say that the running time, $T(n)$, of an algorithm is "in omega of f of n" iff there exist an integer n_0 and a real number $c > 0$ such that $T(n) \geq c \cdot f(n)$ for all $n \geq n_0$.

O

In English

$O(1)$	"constant"
$O(\log n)$	"logarithmic"
$O(n)$	"linear"
$O(n \log n)$	"supralinear"
$O(n^2)$	"quadratic"
$O(n^c)$	"polynomial"
$O(c^n)$	"exponential"
$O(n!)$	"factorial"

Searching



Linear Search

Pseudocode

```
On input n:
    For each element i:
        If i == n:
            Return true.
    Return false.
```

Binary Search

Iterative Pseudocode

```
On input array[0], ... , array[n – 1] and k:
    Let first = 0.
    Let last = n – 1.
    While first <= last:
        Let middle = (first + last ) / 2.
        If k < array[middle] then let last = middle – 1.
        Else if k > array[middle] then let first = middle + 1.
        Else return true.
    Return false.
```

Sum Looping

Get it?

```
int
sigma(int m)
{
    int i, sum = 0;

    /* avoid risk of infinite loop */
    if (m < 1)
        return 0;

    /* return sum of 1 through m */
    for (i = 1; i <= m; i++)
        sum += i;
    return sum;
}
```

see
`sigma1.c`

Sum Recursion

Get it yet?

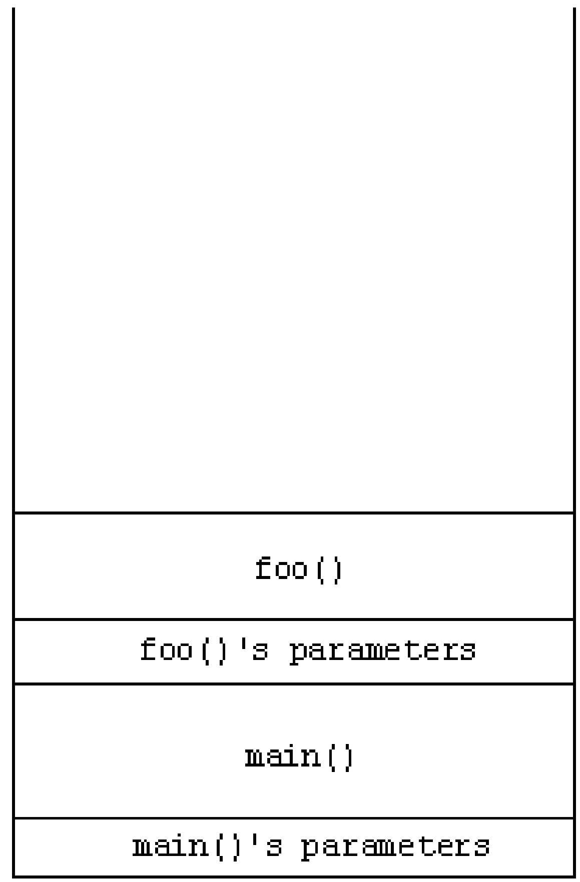
```
int
sigma(int m)
{
    /* base case */
    if (m <= 0)
        return 0;

    /* recursive case */
    else
        return (m + sigma(m-1));
}
```

see
sigma2.c

The Stack, Revisited

Frames



Binary Search

Recursive Pseudocode

```
On input array, first, last, and k, define recurse as:
    If first > last then return false.
    Let middle = (first + last) / 2.
    Else if k < array[middle] then
        return recurse(array, first, middle - 1, k).
    Else if k > array[middle] then
        return recurse(array, middle + 1, last, k).
    Else return true.
```

Sorting

4 2 6 8 1 3 7 5

Bubble Sort

Pseudocode

```
Repeat n times:
    For each element i:
        If element i and its neighbor are out of order:
            Swap them.
```

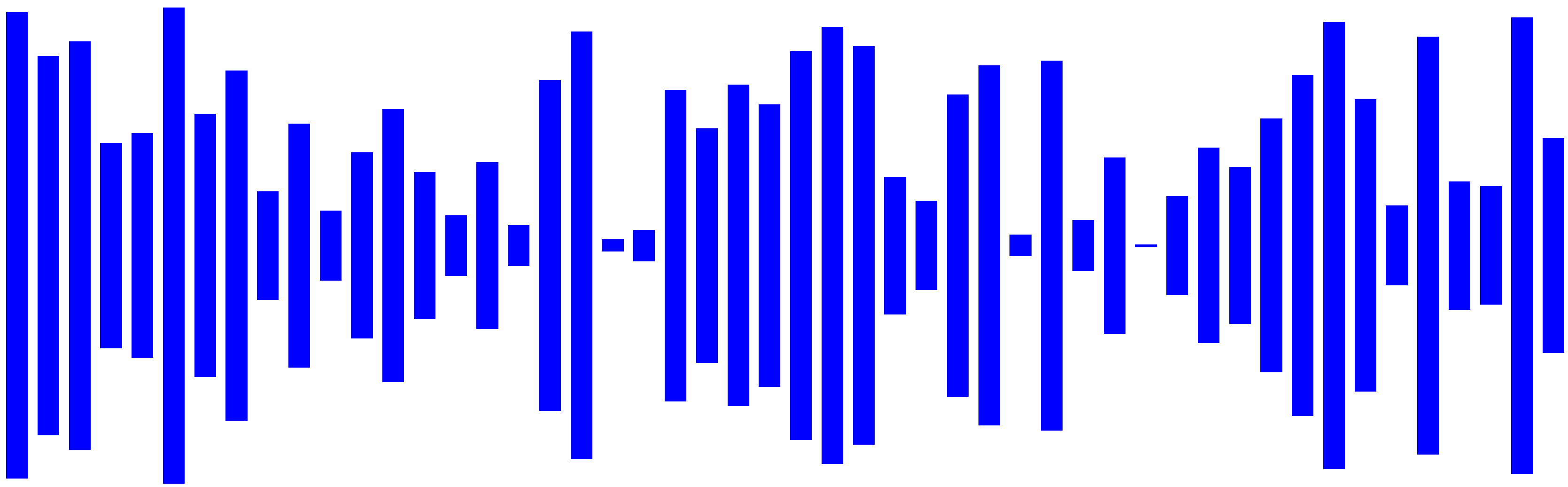
Selection Sort

Pseudocode

```
Let i := 0.
Repeat n times:
  Find smallest value, s, between i and list's end, inclusive.
  Swap s with value at location i.
  Let i := i + 1.
```

Sorting

Visualization



Merge Sort

Pseudocode

```
On input of n  elements:
    If n  < 2, return.
    Else
        Sort left half of elements.
        Sort right half of elements.
        Merge sorted halves.
```

Merge Sort

Pseudocode

$$T(n) = 0, \text{ if } n < 2$$

$$T(n) = T(n/2) + T(n/2) + O(n), \text{ if } n > 1$$

Merge Sort

In the worst case,

how long does it take to sort 16 elements?

Merge Sort

$$
\begin{aligned}
T(16) &= 2T(8) + 16 \\
T(8) &= 2T(4) + 8 \\
T(4) &= 2T(2) + 4 \\
T(2) &= 2T(1) + 2 \\
T(1) &= 0
\end{aligned}
$$

Merge Sort

$T(1)$	$= 0$			
$T(2)$	$= 2T(1)$	$+ 2$	$= 0 + 2$	$= 2$
$T(4)$	$= 2T(2)$	$+ 4$	$= 4 + 4$	$= 8$
$T(8)$	$= 2T(4)$	$+ 8$	$= 16 + 8$	$= 24$
$T(16)$	$= 2T(8)$	$+ 16$	$= 48 + 16$	$= 64$

Sorting

Visualization

All Sorts of Sorts

Heap Sort

Insertion Sort

Quicksort

Radix Sort

Shell Sort

...

Tradeoffs

Space, Time, ...

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