Binary Search
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• In binary search, the idea of the algorithm is to divide and conquer, reducing the search area by half each time, trying to find a target number.
  • In order to leverage this power however, our array must first be sorted, else we cannot make assumptions about the array’s contents.
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In pseudocode:

• Repeat until the (sub)array is of size 0:
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• **Worst-case scenario**: We have to divide a list of n elements in half repeatedly to find the target element, either because the target element will be found at the end of the last division or doesn’t exist in the array at all.

• **Best-case scenario**: The target element is at the midpoint of the full array, and so we can stop looking immediately after we start.
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$O(\log n)$

$\Omega(1)$