Cryptography

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- We ordinarily do this to provide a basic level of security against an adversary who might do bad things with the information, had they been able to see it "in the clear."

Ciphers

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- A wide variety of different types of ciphers exist, with varying levels of inherent security potential.

Substitution Cipher

- Imagine having possession of this device.

Image source: eBay

Substitution Cipher

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- 3 = L
- 4 = M
- 5 = K
- 6 = W
- 7 = N
- 8 = O

Image source: eBay

Substitution Cipher

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Substitution Cipher

- What's the problem with this cipher? Put another way, what is the "attack vector"?

- If the adversary is also a member of Little Orphan Annie's Secret Society, they know how to crack the code.

- We might think of this decoder pin as a "key".

Image source: eBay

Substitution Cipher

- Alternatively, we could use the ordinal positions of letters in a cipher, perhaps.

A	B	C	D	E	F	G	H	I	J	K	…	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	…	24	25	26

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A	B	C	D	E	F	G	H	I	J	K	…	X	Y	Z
3	4	5	6	7	8	9	10	11	12	13	…	26	27	28

+2

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3	4	5	6	7	8	9	10	11	12	13	…	26	1	2

+2

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A	B	C	D	E	F	G	H	I	J	K	…	X	Y	Z
21	22	23	24	25	26	1	2	3	4	5	…	18	19	20

+20

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Caesar Cipher

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- This *rotational cipher* is a rather famous one called Caesar cipher, attributed to Julius Caesar.
- In ancient times, it was apparently very challenging to crack. Nowadays, it's quite easy.
- Limited number of rotational "keys". Only 26 ways to lay the alphabet out.

Vigenere Cipher

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Vigenere Cipher

- The Vigenere cipher is an extended idea to the Caesar cipher, but instead of using a single key, it uses multiple keys, by selecting a keyword.

- Each new letter of the message we want to encrypt (aka the *plaintext*) is enciphered using a different letter of the keyword.

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext					
ordinal position					
keyword					
keyword ordinal position					
sum					
sum, wrapping around					
ciphertext					

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**				
ordinal position	8				
keyword					
keyword ordinal position					
sum					
sum, wrapping around					
ciphertext					

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**				
ordinal position	8				
keyword	**L**				
keyword ordinal position	12				
sum					
sum, wrapping around					
ciphertext					

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**				
ordinal position	8				
keyword	**L**				
keyword ordinal position	12				
sum	**20**				
sum, wrapping around	20				
ciphertext	**T**				

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**	**E**			
ordinal position	8	5			
keyword	**L**	**A**			
keyword ordinal position	12	1			
sum	**20**	**6**			
sum, wrapping around	20	6			
ciphertext	**T**	**F**			

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**	**E**	**L**		
ordinal position	8	5	12		
keyword	**L**	**A**	**W**		
keyword ordinal position	12	1	23		
sum	**20**	**6**	**35**		
sum, wrapping around	20	6	9		
ciphertext	**T**	**F**	**I**		

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- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**	**E**	**L**	**L**	
ordinal position	8	5	12	12	
keyword	**L**	**A**	**W**	**L**	
keyword ordinal position	12	1	23	12	
sum	**20**	**6**	**35**	**24**	
sum, wrapping around	20	6	9	24	
ciphertext	**T**	**F**	**I**	**X**	

Vigenere Cipher

- Let's encrypt the message **HELLO** using the keyword **LAW**.

plaintext	**H**	**E**	**L**	**L**	**O**
ordinal position	8	5	12	12	15
keyword	**L**	**A**	**W**	**L**	**A**
keyword ordinal position	12	1	23	12	1
sum	**20**	**6**	**35**	**24**	**16**
sum, wrapping around	20	6	9	24	16
ciphertext	**T**	**F**	**I**	**X**	**P**

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- So, **HELLO** becomes **TFIXP**.

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- Let's encrypt the message **HELLO** using the keyword **LAW**.
- So, **HELLO** becomes **TFIXP**.
- Unlike Caesar, which is limited to 26 keys, Vigenere cipher has 26^n keys, where n is the length of the keyword chosen.

Substitution Cipher

- Let's assume that your adversary *isn't* a member of Little Orphan Annie's Secret Society.

Image source: eBay

Substitution Cipher

- Let's assume that your adversary *isn't* a member of Little Orphan Annie's Secret Society.

- How might they nevertheless crack a code enciphered with the pin?

Image source: eBay

Frequency Analysis

A	B	C	D	E	F	G	H	I	J	K	L	M
8.1%	1.5%	2.8%	4.3%	12.7%	2.2%	2.0%	6.1%	7.0%	0.2%	0.8%	4.0%	2.4%

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
6.7%	7.5%	1.9%	0.1%	6.0%	6.3%	9.1%	2.8%	1.0%	2.4%	0.2%	2.0%	0.1%

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6.7%	7.5%	1.9%	0.1%	6.0%	6.3%	9.1%	2.8%	1.0%	2.4%	0.2%	2.0%	0.1%

- It's probably tedious for a human to analyze, but a computer can do it very quickly!

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- There are also *transposition* ciphers, which algorithmically rearrange the letters in a message.
- The problem is that all of these classic ciphers are easily cracked, and often suffer from a problem of how to distribute the key.

RECOVER PASSWORD

Enter your email and we'll email you a link to change your password.

Enter Email

SEND PASSWORD LINK SIGN IN REGISTER

Image source: fender.com

Hashes

- A major distinction between ciphers and hashes are that ciphers are (generally) reversible, while hashes are (generally) not.

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- A major distinction between ciphers and hashes are that ciphers are (generally) reversible, while hashes are (generally) not.

- To *hash* some data, we run it through a *hash function*, which mathematically manipulates it in some way, and it outputs a value (sometimes a number, sometimes a string).

Hashes

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- This is why those services can't just tell you what your password is – they don't know it either (hopefully!)

Hash Function

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  - **BBBBBBBBBBBBBBBBBBBBBBBBBBBBB**

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- By running the actual password through the hash function, if we get a match, odds are they entered the right password.

Modern Cryptography

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- Typically the algorithms that are used do not work on a character-by-character basis.
- Most work by taking data of some arbitrary size, and mapping it to a string of bits that is always exactly the same size.

Cryptographic Hash Functions

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  - Be extremely difficult (to the point of infeasibility) to reverse

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  - Never allow two different sets of data to hash to the same value

- The output of a cryptographic hash function is normally referred to as the digest.

SHA-1

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- It works by mapping messages of arbitrary size into a "bit string" of 160 bits. This means that there are 2^{160} different SHA-1 digests, or a bit over 10^{48}.
- It is such an important algorithm that federal regulations require its use.

SHAttered

https://shattered.io/

SHAttered

"It is now practically possible to craft two colliding PDF files and obtain a SHA-1 digital signature on the first PDF file which can also be abused as a valid signature on the second PDF file. For example, by crafting the two colliding PDF files as two rental agreements with different rent, it is possible to trick someone to create a valid signature for a high-rent contract by having him or her sign a low-rent contract."

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- SHA-2, SHA-3
- *MD5*, MD6

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- Email
- Secure web browsing
- VPN
- Document storage
- …

Public-Key Cryptography

- Let's take a trip down memory lane…

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14 x 8 = 112

Public-Key Cryptography

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$$112 \times \tfrac{1}{8} = 14$$

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112 / 8 = 14

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- In this sense, we can think of multiplication as a reversible function. If we multiply some number *x* by some other number *y*, we get a result, *z*.

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- In this sense, we can think of multiplication as a reversible function. If we multiply some number *x* by some other number *y*, we get a result, *z*.

- If we multiply that result, *z* by the reciprocal of *y*, we get back the original *x*. As long as you know what *y* is, it's reversible.

Public-Key Cryptography

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$$14 \times 8 = 112$$

$$f(n) = n \times 8$$

Public-Key Cryptography

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$$f(14) = 112$$

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- Let's take a trip down memory lane…

$$f(14) = 112$$

- Here, imagine *14* is the plaintext and *112* is the ciphertext.

Public-Key Cryptography

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$$f(14) = 112$$
$$f(n) = n \times 8$$

Public-Key Cryptography

- Let's take a trip down memory lane…

$$f(14) = 112$$

$$f(n) = (n \times 10) - 28$$

Public-Key Cryptography

- Let's take a trip down memory lane…

$$f(14) = 112$$

$$f(n) = (4/7) \times n^2$$

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- Two functions, f(n) and g(n), each of which is a one-way function.

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- This is the basic idea behind how public-key cryptography works.
- Two functions, f(n) and g(n), each of which is a one-way function.
- One of those functions is *public* and anyone can use it to encrypt information intended for you. The other is *private*, known only to you, and can be used to reverse the encryption of the first.

Asymmetric Encryption

- To generate these keys, start with a really huge, normally prime, randomly-generated number.

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- To generate these keys, start with a really huge, normally prime, randomly-generated number.
- From there, two complementary one-way functions (quite a bit more complicated than our f(n)) are generated to create a public-private key pair.
- Typically done by a program called RSA.

Asymmetric Encryption

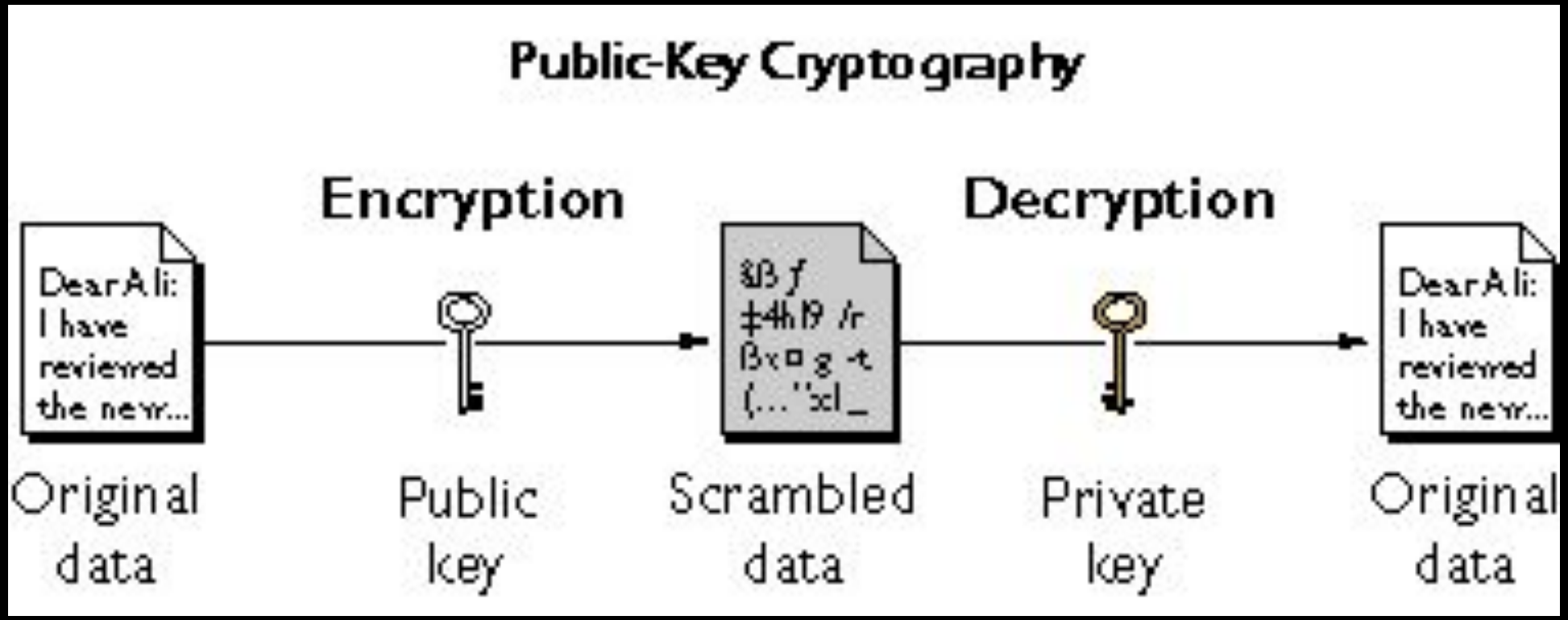

Image source: ibm.com

Asymmetric Encryption

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- The encryption step can be done by anyone who has access to the public key.

- The decryption step can (theoretically) be done only by the individual(s) who have the private key.

Digital Signatures

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- Using a digital signature, one can verify the authenticity of the sender of a document.
- Many digital signatures are 256-bits (meaning 2^{256} distinct digital signatures are possible, meaning the likelihood of a "forgery" is infinitesimal).

Digital Signatures

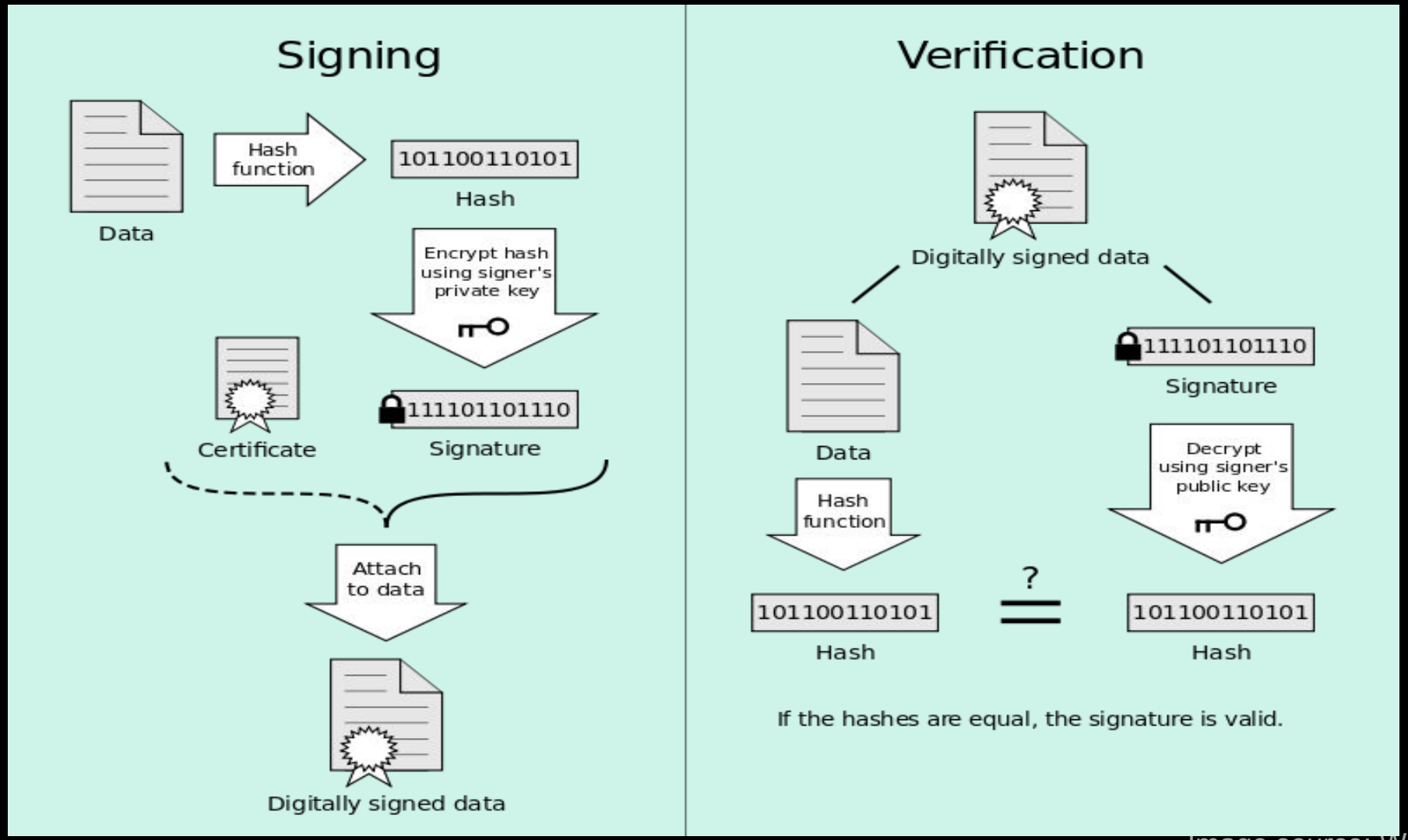

Image source: WikiMedia

Blockchain

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- 3Blue1Brown: https://www.youtube.com/watch?v=bBC-nXj3Ng4

Blockchain

Image source: bitcoin.it

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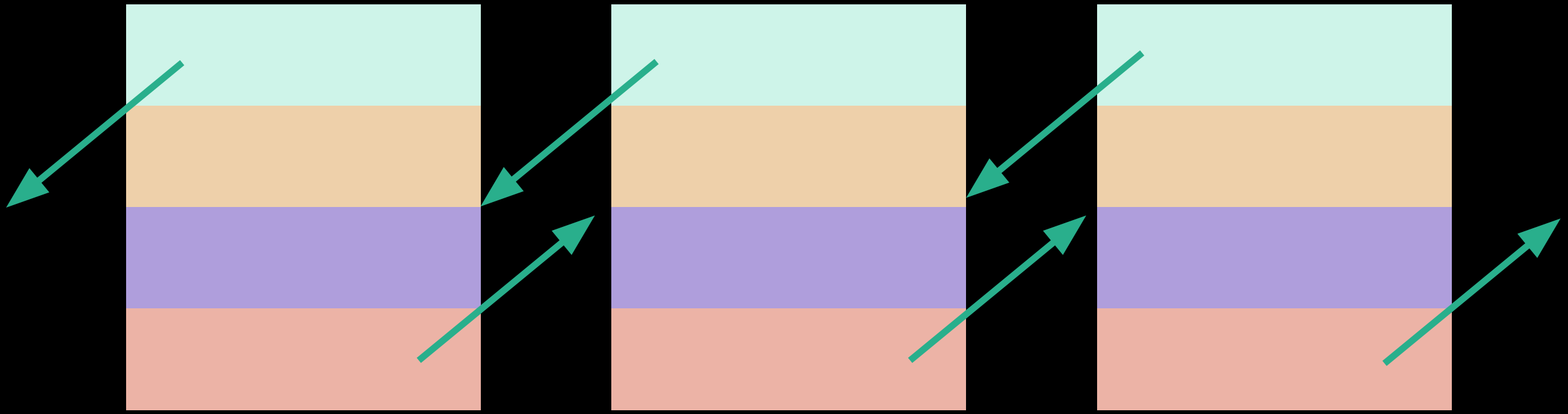

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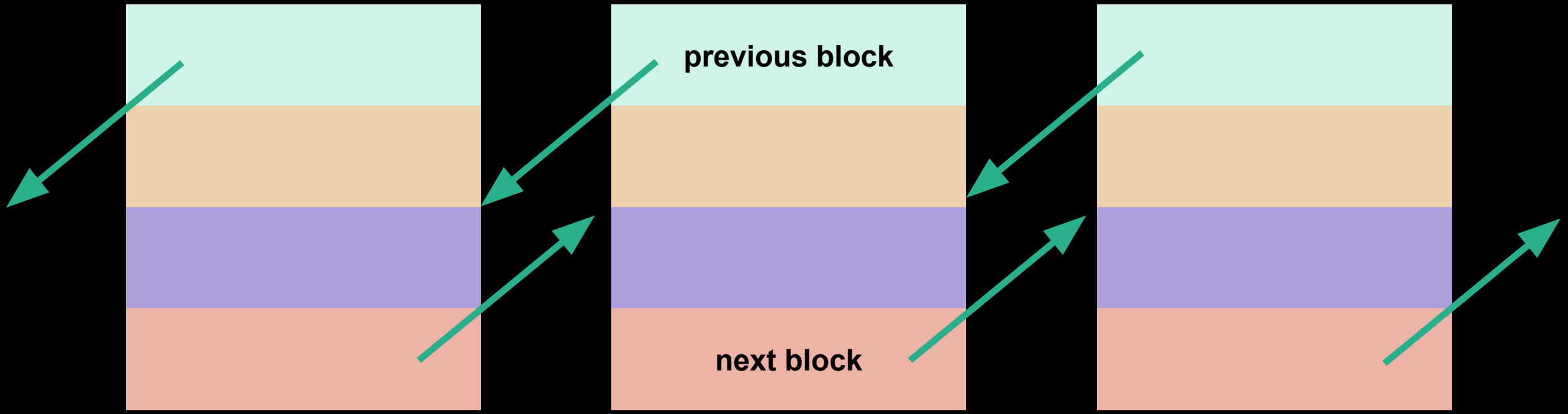

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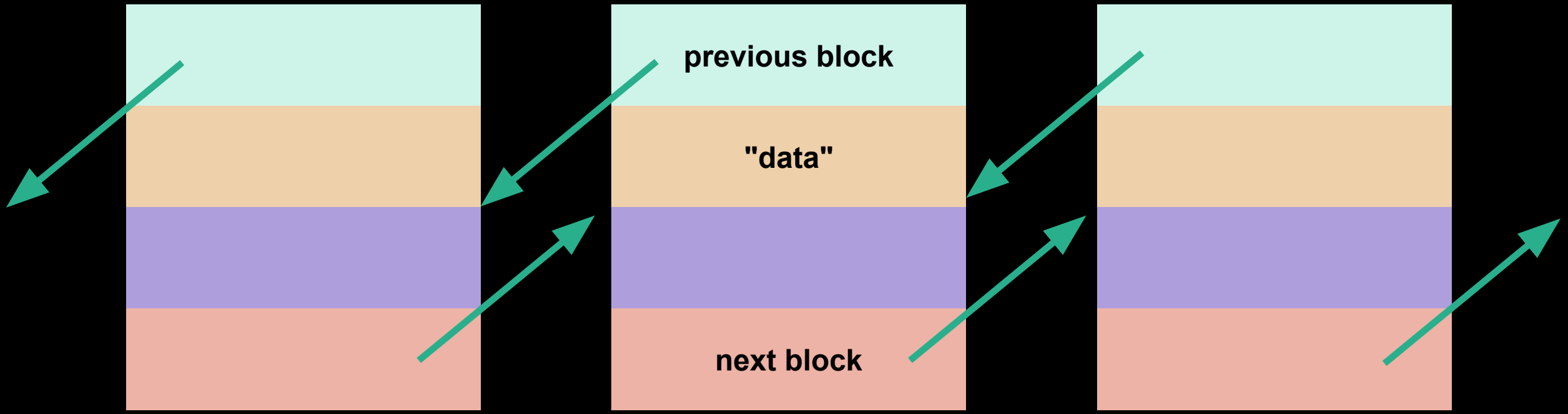

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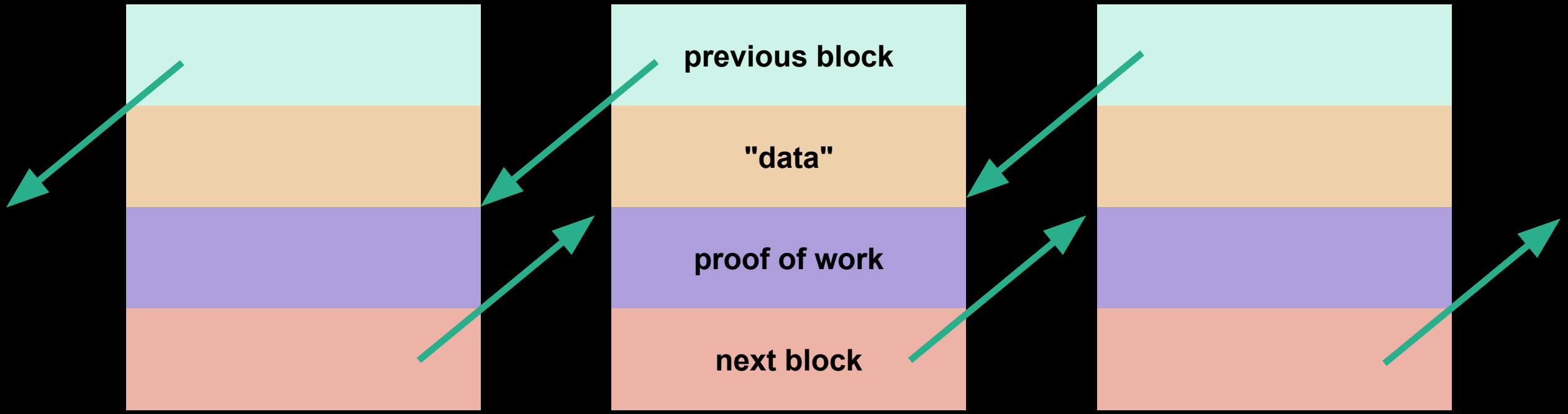

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- It's easiest to think of the blockchain as a linked list.
- What's the data? If we're talking about a cryptocurrency, it's a ledger of transactions, each of which is digitally signed by the person who made that transaction.
- That ledger is also, in the case of a cryptocurrency, *decentralized*, so any time the data is recorded, everyone must record that transaction on their own copy of the ledger, in that block.

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- The way many cryptocurrencies do it is to assume the blockchain with the most computational work put into it is the "true" chain.

- This leads to the concept of proof of work.

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- We can hash the block, over and over, coupled with some random number, until we find a highly unusual pattern in the first *n* (say, 30 to 40) out of 256 bits.

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- Recall how hashing works.
- We can hash the block, over and over, coupled with some random number, until we find a highly unusual pattern in the first *n* (say, 30 to 40) out of 256 bits.
- The smallest change in any of the transactions would produce a totally different hash, making that block unverified (and everything after it potentially fraudulent too.)

Blockchain

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- The longer a chain gets (in the case of chain conflicts), the more and more likely it is that chain consists only of verified, legitimate transactions.

Blockchain

- We can very easily verify the correctness of someone's proof of work.
- The longer a chain gets (in the case of chain conflicts), the more and more likely it is that chain consists only of verified, legitimate transactions.
- What's a transaction?

Blockchain

- What if the data, instead of being a list of transactions, was something else instead?
  - This is what underpins another blockchain-based technology, Ethereum.

Cryptography